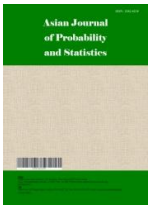

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Bayesian Regression of Government Expenditure on Revenue in Nigeria

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

The relationship between government expenditure and its revenue is generating serious debate among researchers. Similarly, their has been a controversy between the classical and the bayesian modelling. Therfore, this study examined the relationship between the government expenditure and its revenue in Nigeria using the bayesian approach. The finance data extracted from the Central Bank of Nigeria statistical bulletin from 1989 to 2018 were considered for the study. Bayesian linear regression was used to fit the model. Normal distribution was fit for the likelihood. Thus, normal-gamma prior was elicited for the bayesian regression parameters. The result showed that the Bayesian estimates with elicited normal-gamma prior produced a better posterior mean of 0.536 for the Total Revenue with a smaller posterior standard deviation of 0.00001 when compared with the OLS standard deviation of 0.05256. Similarly, the total revenue explained 78% variations in the Total expenditure. The constructed model fit was: Total Expenditure = 98.57128 + 0.53630* Total Revenue. This showed that a naira unit of the total expenditure will always be increased by 0.54 of the total revenue. Forecast of 30 years for the total expenditure using both OLS and Bayesian (normal gamma prior) were increasing as the years were progressing. Government should look for a way to increase its revenue in order to sustain the future expenses of the government since expenditure increases yearly.

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1 Introduction

Public finance is a field of economics concerned with how government raises money, how that money is spent and the effect of these activities on the economy and on the society in general. Therefore Expenditure and revenue of the country are basic tools in public finance however, in a developing economy like Nigeria, management of moderate deficit financing is tailored toward useful and development oriented projects. This called for the attention of this work on the amount of expenditure and revenue generated in Nigeria over the past years. In majority of the developing countries government is seen as an instrument of change and, hence, the size of government expenditure reveals the magnitude of government involvement in the economy. the shift in role of government from traditional functions such as provision of security, administration and law and order to direct intervention income generating activities like capital investment and distributive role like subsidies and transfers have significantly expanded the scope of governments in many countries across the globe.

The major objectives of government are therefore to promote societal welfare by means of appropriate economic, political, social and legal programs. These programs, however, have led to expansion in public expenditure size particularly in the developing economies like Nigeria with a weak and uncompetitive private sector.

Globally government expenditure has been source of interest to both scholars and macro- economic policy makers due to its effects on the level of growth in an economy. Many political philosophers like HOBBES and LOCKE considered the hypothetical disadvantages of life without government [1]. This must have given government in Nigeria and other developing countries, where market failures and other socially unwarranted vices are predominant, the impetus to exercise greater controls and discretion over their economics. Government expenditure has become an important factor for self-sustaining productivity improvements and long term growth.

Sustained and equitable economic growth is clearly a predominant objective of government expenditure policy. It is therefore incumbent on government to allocate public spending across various sectors of an economy in order to maximize prospects of achieving its growth and development objectives.

1.1 Background of the study

In Nigeria government expenditure can be described as expenses which any government for its own maintenance, for the good of society and the economy, and for assistance to external bodies and other countries [2]. It refers to the expenditure of government on governmental bodies and on various segments of the economy. A good pattern of government expenditure encourages economic growth, favors the provision of employment and good roads, and ensures increase in salaries of civil servants. Government expenditure pattern in Nigeria has been complained of by some Nigerians, through the TV and print media, as not having the desired effect on the economy. It was that more money is allocated to the executive arm of government and that corruption adversely effects government expenditure. Also, they complain that their social amenities needs are not met. Added to this, it is not just that the major part of the government expenditure goes into consumption, a substantial amount of money budgeted for capital expenditure, at the presence of fraud and embezzlement of public funds in Nigeria goes into corruption [3-5].

Expenditure refers to payments made or liabilities incurred in exchange for goods or services, it is usually a one-time cost and is incurred to receive a long term benefit, such as the purchase of fixed asset, the three types of expenditure are capital expenditure, revenue expenditure, and deferred revenue expenditure. Capital expenditure is a one-time cost, benefit of which is expected to be spread over multiple years. revenue expenditures are usually recurring expenses, the benefit of which are received during the accounting year. they can be either direct or indirect expenses [6-9]. Deferred expenditure refers to an advance payment for goods or services, the benefit of which is to be received only in the future.

Government expenditure includes all government consumption, investment, and transfer payments. Government acquisition of goods and services intended to create future benefits, such as infrastructure investment or research spending, is classified as government investment, purpose of government expenditure is to achieve improvements in the supply-side of the macro- economy, such as spending on education and training to improve labour productivity [10-12].

Government revenue is money received by a government from taxes and no-tax sources to enable it to undertake government expenditure. It is a components of the government budget and important tools of the government fiscal policy , the government revenues earned by the government are obtained from different sources such as taxes leveled on the incomes and wealth accumulation of individuals and corporations on the goods and services produced, exports and imports, non taxable sources such as government owned corporations' incomes, central bank revenue and capital receipts in form of external loans and debts from international financial institutions government use revenue to better develop the country to fix road, build homes, fix schools etc [13,14]. the money that government collects pay for services that are provided for the people. Every government depends on different sources to thrive and provide services to her citizens generally government revenue can be divided into oil and non-oil revenue [15,19].

The non-oil revenue includes proceeds from federal taxes and levels, agricultural sector, aviation, federal exam fees, various forms of license fees, import duties and tariffs. The oil revenue is simply all proceeds from the oil sectors, they are proceeds from oil sales, and remittances from international and national oil companies. This is the major source of revenue for the federal government.

2 Literature Review

Onifade and Cevik [20] studied an empirical retrospect of the impacts of government expenditures on economic growth, pesaran's ARDL Approach has been applied to carry out the impact analysis using annual time-series data from 1981 to 2017. Empirical findings support the existence of a level relationship between public spending indicators and economic growth in Nigeria, recurrent expenditures of government were found to be significantly impacting on economic growth in a negative way while the positive impacts of public capital expenditures were not significant to economic growth over the period of the study, results from granger causality test reveal that fiscal expansion of the government that is hinged on debt financing is strongly granger causing public expenditures and domestic investment with the latter also granger causing real growth in the economy.

Anthony and Tope [21] studied the impact of tax revenue on nigeria economy, descriptive survey design was adopted and simple random sampling technique was used in the selection of the sample size. Four hypotheses were formulated and tested using chi-square statistical tool of analysis, the findings show that tax revenue significantly impact on federal government budget implementation in Nigeria, tax administrative system significantly affected the revenue generated in Nigeria, tax evasion significantly affected the revenue in Nigeria, and lack of training on the part of tax officers significantly affected the generation of government revenue in Nigeria.

Adamu and Chandana [22] studied modelling the determinants of government expenditure in Nigeria, the study employs a slightly modified version of Wagner's law by incorporating new variables such as oil revenue, trade openness, public dept, exchange rate ,oil price, taxation and inflation- to examine their effect on government expenditure size, time series data were analyzed using .autoregressive distributed lag (ARDL) model, the findings of the study reveals that oil revenue, GDP, population, trade openness, oil price, taxation and inflation are important determinants of the size of Nigeria's government expenditure.

Bonmwa and Ogburu [23] this research examined the impact of government expenditure and economic growth in Nigeria from 1981 to 2016, the impact of government recurrent and capital expenditures were tested using two separate models. Stationarity of the variables were tested to determine the stochastic properties of the series. Also, the co-integration result indicates that the two models each have one co-integrating equation. An ordinary least squares technique with error correction specifications was used to analyze the data. The result for the model 1 indicates that the coefficients of social and economic services were negative while administration and social services were negative and insignificant while economic services were positive but significant. The result

for model 2 indicates that coefficients of administration and social services were negative and significant. The study therefore concluded that government expenditure has not translated into meaningful economic growth.

Oladele, Giseleah and Itumeleng [24] this study examine the contribution of government spending towards economic growth in south Africa using annual data from 1980-2014. The co-integration approach and vector correction model were used to analyze the data. The co-integration test results indicate that there is long run relationship between government expenditure and economic growth in South Africa. The VCEM outcome indicates a positive and significant link between economic growth and expenditure on the long run. There is positive and significant relationship between exchange rate and economic growth and private consumption, based on the findings, the correlation between government expenditure and economic growth showed that there is positive relationship on the long run in south Africa, while there is negative and significant relationship between government spending and economic growth on the short run.

Uguru (2016) empirically examined the relationship between public dept and government expenditure in Nigeria from 1980-2013.the data used was purely secondarydata sourced from central bank of Nigeria statistical bulletin for various years. The study employed the ordinary least square regression technique and found that there is a significant relationship between public debt and government expenditure in Nigeria.

Agbonkhese and Asekome (2014) studied the impact of public expenditure on the growth of Nigerian economy from 1981-2011. They employed ordinary least square (OLS) method of econometric technique and found that although there is a positive relationship between the dependent and independent variables, the adjustment of economic growth or gross domestic product was a fair one which made it difficult to reject the null hypothesis which according to them implies that government over the years appears to be bad managers of resources and have failed to play their role in the process of economic growth and development.

Ogujuiba and Abraham (2014), studied testing the relationship between government revenue and expenditure, the paper examines the revenue- spending hypothesis for Nigeria using macro data from 1970 to 2011, correlation analysis, granger causality test, regression analysis, lag regression model, vector error correlation model and impulse response analysis were the techniques used for analysis, the paper found that revenue and expenditure are highly correlated and that causality runs from revenue to expenditure in nigeria. The VECM also confirms that there is a significant long run relationship between revenue and expenditure implying that disequilibrium in expenditure can be corrected in the long run through policies that adjust oil and non-oil sector revenues. The lagged regression model showed that the positive relationship between revenue and expenditure. The paper concludes that short term shocks from crude oil price passes through oil revenue to affect expenditure.

Ali and Shan (2012) in Pakistan, who examined government revenue and expenditure nexus using annual data for the period 1976-2009 they applied the Johansen co–integration and granger causality techniques and found no relationship among the variables both in the long run and the short run granger. This result supports institutional separation hypothesis.

Aregbeyan and Ibrahim (2012) examined the long run relationship and the dynamic interaction between government revenues and expenditures in Nigeria from 1970-2008 using Autoregressive Distributed Lag (ARDL) bound test approach. The paper introduced an innovation that examined the relationship of revenue and expenditure but did not test their direction of influence. Thus, it is not clear why revenue or expenditure was specified bas dependent or independent variable.

Elyasi and Rahimi [25] also investigated the relationship between government revenue and expenditure in Iran by applying the bounds testing approach to co integration they showed that there is a bidirectional causal relationship between government expenditure and revenues in both the long run and short run.

Ogujiuba and Abraham [26] also examined the revenue- spending for Nigeria using macro data from 1970 to 2011 applying correlation analysis, granger causality test, regression analysis, lag regression model, vector error correction model and impulse response analysis, they report that revenue and expenditure are highly correlated and that causality runs from revenue to expenditure in Nigeria. The vector error correction model also proves that there is a significant long run relationship between revenue and expenditure.

Saeed and Somaye(2012) investigated the causality and the long run relationships between government expenditure and government revenue in oil exporting countries during 2000-2009 using P-VAR frame-work and considering the oil revenue as proxy for total revenue. Their results revealed that there is positive unidirectional long-run relationship between oil revenue and government expenditures.

Ebringa, Oforegbunam Thaddeus (2012) studied the impact of government sectorial expenditure on the economic growth of Nigeria usinga Cochrane-orcutt and ECM method to measure the long run effect of selected macroeconomic variables economic growth. The results shows that expenditure on telecommunication, defense and security, education and health sector have made positive impact on Nigeria's economic growth. But transportation and agricultural expenditures have impacted negatively in the economic growth in nigeria. The conclusion therefore is that the level of government expenditures for transportation and agricultural development is still not adequate to build the muchneed capacity in the sectors to impact positively to economic growth.

Emelogu and Uche (2010) studied the relationship between government revenue and expenditure in Nigeria using time series data from 1970 to 2007. They utilized the engel-granger two step co-integration techniques, the johansen co–integration method and granger causality test within the ErrorCorrectionModeling (ECM) framework. It was found that a long –run relationship between the two variables and a unidirectional causality running from government revenue in Nigeria.

Abu and Abdullahi (2010) examined government expenditure and economic growth in Nigeria from 1970-2008.empowering ordinary least squares (OLS) method they found that government capital and recurrent expenditure have negative and non-significant effect on economic growth of Nigeria.

Mehmood and Sadiq [27] examined the short and long run relationship between the fiscal deficits and poverty in Pakistan using time series data from 1976-2010. The short run relationship was examined using error correction model while johensen co integration analysis was used to examine the long run relationship. The paper found that there is negative relationship between government expenditure both in short and long run in Pakistan. Though the authors used johansen co integration model to estimate the long run relationship using a single framework. Normality test should have been added to the stationarity test in order to avoid serious regression results.

Hye and Jalil [28] in Romania adopted the autoregressive distributive lag approach to co integration, variance decomposition and rolling regression method to determine the causal relationship between expenditure and revenue of government the results indicate that bidirectional long run relationship exist between government expenditure and revenue. The variance decomposition results further suggest that government revenue block has sharp impact on the government expenditure compared to the revenue collection response to shock in government expenditure.

Zinaz and Samina (2010) employed granger causality test on a bivariate model to study the causality between government expenditure and tax revenue. They conclude that there exists a unilateral stable long run relationship running from expenditure to revenues in Malaysia.

OlaoluOlayeni [29] studied a Bayesian analysis of government expenditure in Nigeria, the paper examines the productivity of government expenditure. It adopts aBarro-type production function to chart out a growth model that accounts for the productivity of government spending and also adopts wagner's hypothesis to account for endogeneity resulting from fiscal expansion. The model is estimated via the Bayesian technique using the data on Nigeria. The results shows that government expenditure was unproductive in Nigeria and that this conclusion is independent of the macroeconomic environment.

Tracy and Kester (2009) investigated the relationship between total government expenditure and total tax revenue in Barbados applying granger causality on both bivariate and multivariate co-integrated models. The result of the multivariate error correction model suggests that a unidirectional causality exists from tax revenue to government expenditure.

Stoian [30] examined the relationship between public revenues and expenditures in Romania. Regression analysis, correlation and granger causality analysis were used for the analysis. The results found a significant

relationship between the variables. the paper argued that the direction of causality (that runs from revenue to expenditure) implied that some adjustment are required in revenues to achieve desired targets of expenditures, the paper also argued that expenditures could respond to lagged values of revenues but did not provide empirical evidence in that direction. From granger causality result also, the paper deduced there could be a long run equilibrium between revenue and expenditure that could be reached by through short run adjustments in revenue. Empirical evidence that shows such possibility was not presented in the paper,more so specific economists are likely to respond to shocks from their major exports, like crude oil for Nigeria. Providing evidence on expenditure response to major revenue shock channels would therefore be important as well.

Al-Qudair [31] examined the long run equilibrium relationship between government expenditure and revenues in the kingdom of Saudi Arabia using co integration techniques ECM and granger causality test indicates the existence of long run equilibrium between government expenditure and revenues. The causality tests show the existence of bi-directional causal relationship between government expenditure and revenues in the long and short run.

Moalusi (2004) examined the causal relationship between government spending and government revenue In Botswana. The results of both bivariate and multivariate models provided evidence of a unidirectional causal link running from revenue to spending.

Attamah [32] writes the traditional function of government expenditure is the maintenance of the bureaucratic structure (i.e. the civil service) and defense. Today, governments perform a variety of economic functions, according to him during the industrial revolution; poverty was increasing at an alarming rate, and as an offshoot of the increasing suffering of the laborers'.

Mithani and Khoon [33] incorporated the effect of seasonality to examine the causal relationship between quarterly government revenue and government expenditure in Malaysia between 1970-1997 they report evidence of seasonal error correction model indicates a unidirectional causal influence from government expenditure to government revenue. The implication of this result is that spending decision determines the size and growth of public sector and consequential tax burden as well as fiscal deficit in Malaysia.

Ayanfo (1996) describes expenditure as an actual payment or the creation of an obligation to make a future payment for some benefit, items or service received, Hales (1994) defines expenditure as payment, or promise of future payment and the obligation incurred there under, for goods and services delivered.

Musgrave [34] More so developed the fiscal synchronization hypothesis, states that citizens compare the marginal benefits and marginal costs of government services in making fiscal policy decision. Therefore, it is characterized by bidirectional causality between government revenue and expenditure. Some empirical studies have supported this hypothesis like Baharumshah, Jibrilla, Sirag, Ali and Muhammad (2016) find support for fiscal synchronization hypothesis using data for South Africa. Similar evidence is found by Phiri(2016). Contrary, Ali and Amin (2018) find support for neutrality hypothesis indicating that there is absence of causality between the fiscal variables which signifies that revenue and expenditure are independent of each other.

Peacock and Wiseman [35]; discussed about the 'advanced - spend–revenue hypothesis', that changes in public expenditure brings about changes in public revenue. This hypothesis has been supported by several empirical studies using data for different countries (Narayan and Narayan, 2006; Parida, 2012; Richter andDimitrios, 2013;Saunorisand Payne, 2010; Zapf and Payne, 2009). The spend–tax hypothesis places expenditures ahead of revenues. The effect could be scary if proper policies are not devised to cushion the escalation of budget deficit with consequence of shifting repayment burden on the future tax payers.

3 Methodology

In the Bayesian view point, linear regression is formulated using probability distributions rather than viewpoint estimates. The response y is not estimated as a single value, but is assumed to be drawn from a probability distribution. The model for Bayesian linear regression with response sampled from a normal distribution is $y \sim N\ (\beta^T X, \sigma^2 l)$

The output y is generated from a normal distribution characterized by a mean and variance. The mean for linear regression is the transpose of the weight matrix multiplied by a predictor matrix and the variance is the square of the standard deviation.

A joint posterior distribution of parameters is used to define the 'subjective belief's during calibration the probability function of a particular parameter is expressed using Baye's theorem:

$$P(X \mid Y = \text{data}) = \frac{P(Y \mid X)P(X)}{P(Y)} \quad (1)$$

$$p(parameters \mid Y) = \frac{p\left(Y \mid parameters\right)p(parameters)}{p(Y)} \quad (2)$$

Data are treated as a fixed set of information to be used in updating ones beliefs about the parameters. In equation (2) above If we ignore the marginal density of the data we have

$$p(parameters \mid y) \propto p(y/parameters)p(parameters) \quad (3)$$

P(parameters/data) is the is the posterior density of the parameters, given the current body of data p(data/parameters) is the joint distribution of the data given the parameters it is also called the likelihood function p(parameters) is the prior density which represent the prior beliefs of the analyst about the parameters. We re-write Equation (3) as

Posterior density ∝ likelihood function × prior density (4)

3.1 Bayesian Approach to Linear Regression

Consider a simple two- variable linear model with dependent variable y_i expressed as linear combination of one explanatory variable plus an error term.

Specifying the model as;

$$Y_i = \beta_o + \beta_1 x_i + U_i \quad (5)$$

Where y_i and $x_i$denotes the observed data on the dependent and explanatory variables respectively, for $i = 1, \dots, n$; u_i is the error term which is independently and identically distributed with N(0, σ_u^2).

3.1.1 Bayesian simple linear regression using reference prior

Theassumption that the errors u_i are independent and identically distributed as normal random variables with mean zero and constant variance $\sigma^2$is exactly the same as in the case of the classical inference for testing and constructing confidence intervals for $\beta_o and\ \beta_1$, in order to update the distributions of the unknown parameters $\beta_o,\ \beta_1 and\ \sigma^2$ based on the data $x_1, y_1, \dots, x_n, y_n$ where n is the number of observations. Based on this assumption, the random variable of each responsey_i, conditioning on the observed data x_i and the parameters $\beta_o,\ \beta_1 and\ \sigma^2$ is normally distributed:

$$y_i \mid x_i, \beta_0, \beta_1, \sigma^2 \sim normal\ (\beta_o + \beta_1 x_i, \sigma^2), \qquad i = 1, \dots, n \quad (6)$$

That is, the likelihood of each y_i given $x_i, \beta_o, \beta_1\ and\ \sigma^2$ is given by

$$p(y_i \mid x_i, \beta_o, \beta_1, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_i - (\beta_o + \beta_i x_i))^2}{2\sigma^2}\right). \quad (7)$$

The likelihood of $y_1, \dots, y_n$ is the product of each likelihood $p\left(y_i \mid x_i, \beta_o, \beta_1, \sigma^2\right) p(y_i \mid x_i, \beta_o, \beta_1, \sigma^2)$, since we assume each response y_i is independent from each other. Since this likelihood depends on the values of

$\beta_o, \beta_1 \; and \; \sigma^2$, it is sometimes denoted as a function of $\beta_o, \beta_1 \; and \; \sigma^2$: $L(\beta_o, \beta_1, \sigma^2)$. using the reference prior which is the standard non –informative prior, posterior distribution of $\beta_0, \beta_1, \sigma^2$ which gives the analoque to the frequentist results, the joint prior distribution of $\beta_o, \beta_1 \; and \sigma^2$ is assumed to be proportional to the inverse of σ^2

$$p(\beta_o, \beta_1, \sigma^2) \propto \frac{1}{\sigma^2}, \tag{8}$$

Using the hierarchical model framework, this is equivalent to Assuming that the joint prior distribution of $\beta_o, \beta_1 \; under \sigma^2$ is the uniform prior, while the prior distribution of σ^2, is proportional to $\frac{1}{\sigma^2}$ that is

$$p(\beta_o, \beta_1 \mid \sigma^2) \propto 1, \qquad p(\sigma^2) \propto \frac{1}{\sigma^2} \tag{9}$$

combining the two using conditional probability to get the same joint prior distribution , then applying baye's rule to derive the joint posterior distribution after observing data $y_1, \ldots, y_n \, y_i, \ldots, y_n$. Bayes rule states that the joint posterior distribution of $\beta_o \beta_o, \; \beta_1 \beta_1 \; and \; \sigma^2 \sigma^2$ is proportional to the product of the likelihood and the joint prior distribution

$$\begin{aligned} p^*\left(\beta_o, \beta_{1,} \sigma^2 \mid y_i, \ldots,\right) &\propto \{n \prod \prod ip(y_i \mid x_{i,} \beta_o, \beta_1, \sigma^2)\} p\,(\beta_o, \beta_1, \sigma^2) \\ &\propto \left\{\left(\frac{1}{(\sigma^2)^{1/2}} \, exp\left(-\frac{(y_i - (\beta_o + \beta_1 x_i))^2}{2\sigma^2}\right)\right\}\right. \\ &\times \ldots \left(\frac{1}{(\sigma^2)^{1/2}} \exp\left(-\frac{(y_n - (\beta_o + \beta_1 x_n))^2}{2\sigma^2}\right)\right) \times \left(\frac{1}{\sigma^2}\right) \\ \propto \frac{1}{(\sigma^2)^{(n+2)/2}} \, exp\left(-\frac{\sum_i (y_i - \beta_o - \beta_1 x_i)^2}{2\sigma^2}\right) \end{aligned} \tag{10}$$

3.1.2 Bayesian simple linear regression using Conjugate priors

Except from the hierarchical reference priors, using semi-conjugate prior distribution of β_o , β_1 and σ^2 is considered where there is information available about the parameters. Since the data $y_1, \ldots, y_n$ are normally distributed, a normal- gamma distribution will form conjugacy in this situation. Prior distribution is set through a hierarchical model. are assumed that given and σ^2 ,β_o and β_1 follow the bivariate normal prior distribution, from which their marginal distributions are both normal,

$$\beta_o \mid \sigma^2 \sim normal\left(a_o, \sigma^2 s_{\beta_o}\right) \tag{11}$$

$$\beta_1 \mid \sigma^2 \sim normal\left(b_o\,, \sigma^2 s_{\beta_1}\right) \tag{12}$$

With covariance

$$cov\left(\beta_o, \beta_1 \mid \sigma^2\right) = \sigma^2 s_{\beta_o \beta_1.} \tag{13}$$

Where, $\sigma^2, s_{\beta_o}, s_{\beta_1} and \; s_{\beta_o \beta_1}$ are hyper -parameters. This is equivalent to setting the coefficient vector $\beta = (\beta_o\,, \beta_1)^T$ to have a bivariate normal distribution with covariance matrix $\sum_o$

$$\sum_o = \sigma^2 \begin{pmatrix} s_{\beta_o} & s_{\beta_o \beta_1} \\ s_{\beta_o \beta_1} & s_{\beta_1} \end{pmatrix} \tag{14}$$

That is,

$$\beta = (\beta_o, \beta_1)^T \mid \sigma^2 \sim bivariatenormal(\boldsymbol{b} = (a_o, b_o)^T, \sigma^2 \textstyle\sum_o) \tag{15}$$

Then for σ^2 , inverse gamma distribution is used as its prior distribution

$$1/\sigma^2 \sim gamma\left(\frac{v_o}{2}, \frac{v_o\sigma_o}{2}\right) \quad (16)$$

3.1.2.1 Joint normal- gamma posterior distributions

Recall that the joint posterior distribution of $\beta_1 \; and \; \sigma^2$ is

$$p\left(\beta_1, \sigma^2 \mid y\right) \propto \frac{1}{\sigma^{2\frac{(n+1)}{2}}} exp\left(-\frac{SSE+(\beta_1-\hat{\beta}_1)^2\sum_i(x_i-\bar{x})^2}{2\sigma^2}\right) \quad (17)$$

This is re-written using precision $\phi = 1/\sigma^2$then the joint posterior distribution of $\beta_1 \; and \; \phi$ will be

$$p\left(\beta_1, \phi \mid y\right) = \phi^{\frac{n-2}{2}} exp\left(-\frac{\phi}{2}\left(SSE + (\beta_1 - \hat{\beta}_1)^2 \sum_i (x_i - \bar{x})^2\right)\right) \quad (18)$$

It can be viewed as the product of the posterior distribution of $\beta_1$conditioning on $\phi$and the posterior distribution of ϕ

$$\pi\left(\beta_i \mid \phi, \; y\right) \times \pi\left(\phi \mid y\right) \propto \left[\phi exp\left(-\frac{\phi}{2}(\beta_1 - \hat{\beta}_1)^2 \sum_i (x_i - \bar{x})^2\right)\right] \times \left[\phi^{\frac{n-2}{2}-1} exp\left(-\frac{\phi}{2}(SSE)\right)\right] \quad (19)$$

The first term in the product is exactly the normal distribution with mean $\hat{\beta}_1$and standard variance

$$\frac{\sigma^2}{\sum_i (x_i-\bar{x})^2} = \frac{\sigma^2}{s_{xx}} \quad (20)$$

$$\beta_1 \mid \sigma^2, y \sim normal\left(\hat{\beta}_1, \frac{\sigma^2}{s_{xx}}\right) \quad (21)$$

The second term, is the gamma distribution of the precision ϕ, or the inverse gamma distribution of the variance $\boldsymbol{\sigma^2}$

$$1/\sigma^2 \mid y \sim gamma\left(\frac{n-2}{2}, \frac{SSE}{2}\right) \quad (22)$$

This means the joint posterior distribution of $\beta_1 \; and \; \sigma^2$, a normal-gamma distribution.

$$\beta_o \mid \sigma^2, y \sim normal\left(\hat{\beta}_o, \sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{s_{xx}}\right)\right) \quad (23)$$

$$1/\sigma^2 \; y \sim gamma\left(\frac{n-2}{2}, \frac{SSE}{2}\right) \quad (24)$$

4 Data Analysis and Discussion of Result

This section shows Bayesian regression result of the data. Total Expenditure is the dependent variable while Total revenue is the independent variable. In the Bayesian regression, normal distribution is used for the likelihood of the data. Normal-gamma and Uniform were elicited for the priors. The data are secondary extracted from the Central Bank of Nigeria Bulletin. It spansthrough 1989 to 2018.

4.1 Descriptive Statistics

Table 1 below shows the descriptive statistics of the variables considered in this study. The table shows the mean, first quartile, median minimum, third quartile and the maximum values.

Table 1. Descriptive Statistics of the variables

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Total Revenue	53.87	478.61	3247.80	4152.02	7205.88	11116.85
Total Expenditure	41.03	359.97	1326.10	2325.29	4489.19	7813.74

Fig. 1 below shows the scatter plots of Total expenditure against Total revenue. It helps to see the kind of relationship that exists between these two variables. From the plot below, it could be observed that there is specific relationship between them as the point is linear. This propels us to using a robust approach in fitting a relationship between Total revenue and Total expenditure. Bayesian regression approach will be used to fit a model between Total expenditure and Total revenue.

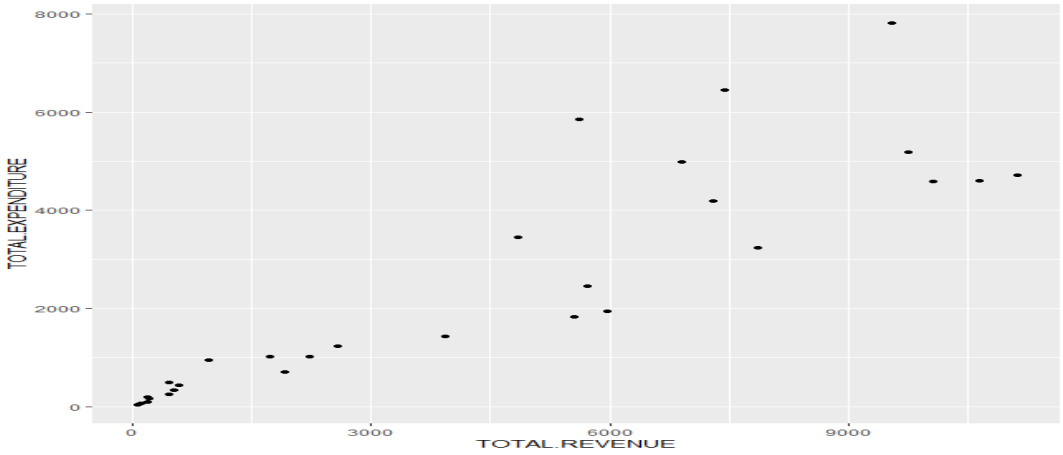

Fig. 1. Scatter plot of Total expenditure against Total Revenue

Fig. 2 shows the density plot for Total expenditure and Fig. 3 shows the boxplot for both the Total expenditure and Total Revenue. The density plot shows a bi-modal plot as two maximum point are displayed in the density plot. The first modal occurs between #0 and #2000 while the second shows between #4000 and #6000.In the box plot, Total Expenditure shows colour red and total Revenue shows colour green. The box plot shows the minimum, first quartile, median, third quartile and the maximum value.

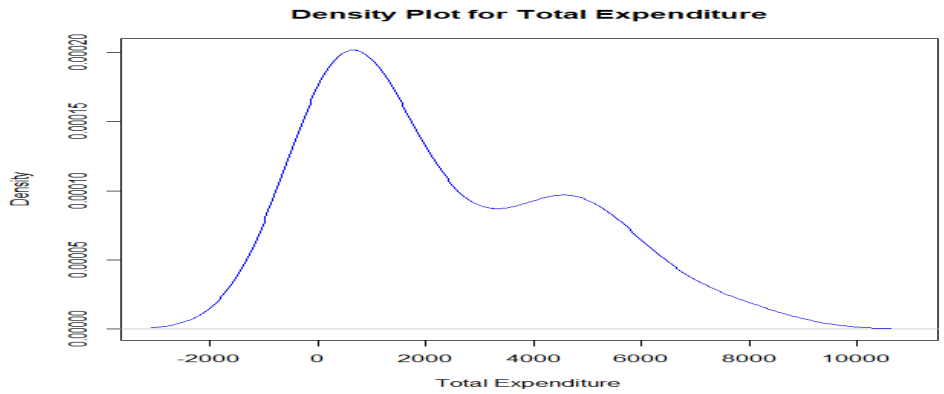

Fig. 2. Density plot for total expenditure

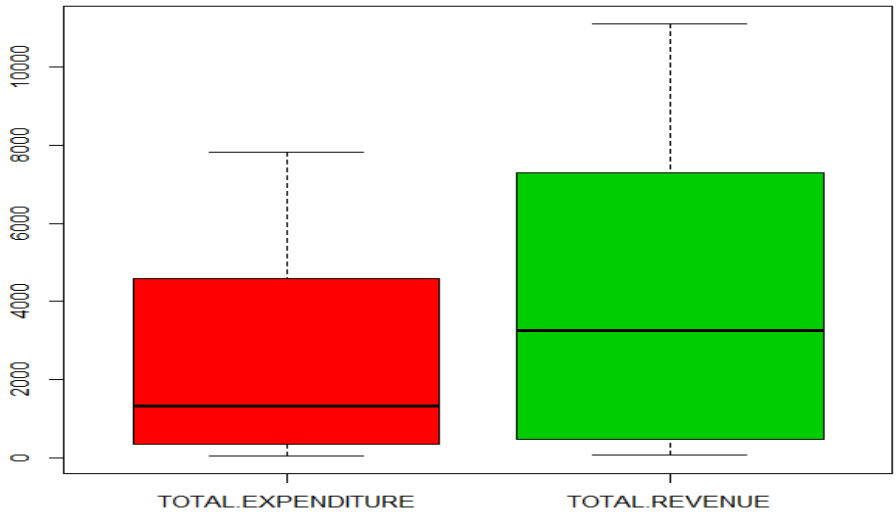

Fig. 3. Box plot for Total expenditure and Total Revenue

4.2 Classical Linear Regression

4.2.1 Ordinary least squares result

Table 2 below shows the estimates of the OLS result. It could be seen that the intercept produces estimate of 98.57128 with standard error 0.28307. The independent variable Total revenue produces the estimate of 0.53630 with standard error of 0.05256. This table depicts, at no contribution of Total Revenue, Total Expenditure will always be 98.57128. Also, at every unit contribution of Total Revenue, Total Expenditure will always be added by the multiple of 0.53630. The model shows a 78% adjusted R-square. This implies that Total revenue explains 78% of the Total expenditure for this study.

Table 2. Coefficients of ordinary least squares

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	98.57128	0.28307	0.336	0.739
Total Revenue	0.53630	0.05256	10.203	6.17e-11 ***

Multiple R-squared: 0.788, Adjusted R-squared: 0.7805

Tables 4 and 5 below show the normal Q-Q plot for the regression and the fitted line plot over the scattered plot using ordinary least square method.

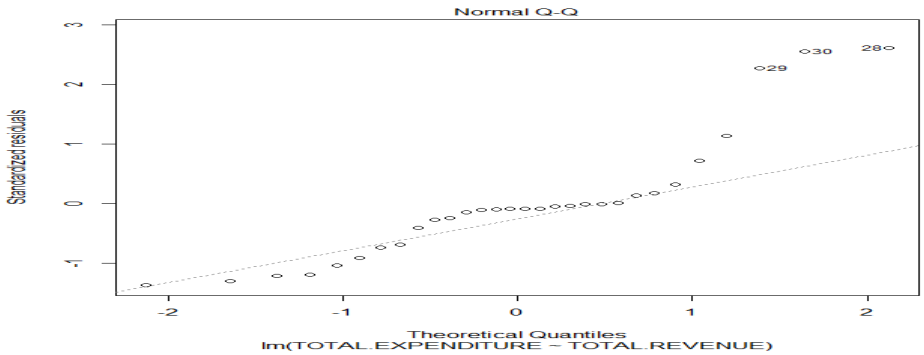

Fig. 4. Q-Q plot of the fitted regression between Total expenditure and Total Revenue

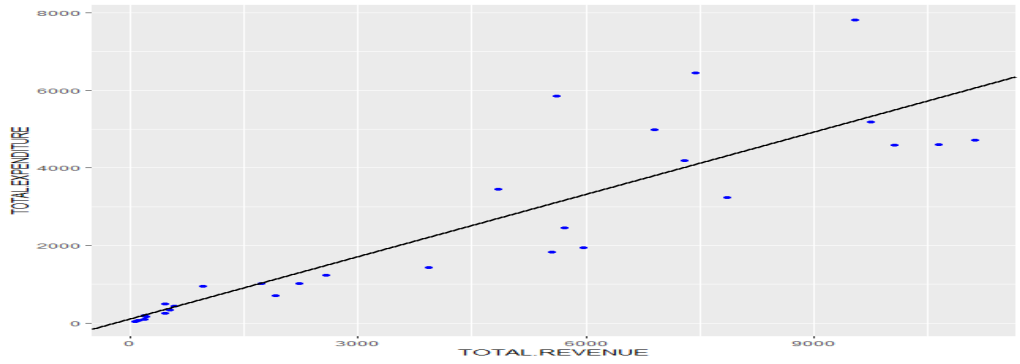

Fig. 5. Regression line on the scatter plot

4.2.2 Bayesian regression results

Bayesian regression was obtained for linear regression using Normal-gamma distribution as priors againstassumed normal distribution as likelihood for the data. The results obtained using the two priors are discussed below.

Table 3 shows 98.57 posterior mean for the intercept with posterior standard deviation of 0.061 while the independent variable Total revenue produces a posterior mean of 0.536 with posterior standard of 0.00001. The credible interval between 2.5% and 97.5% was also displayed in the table.

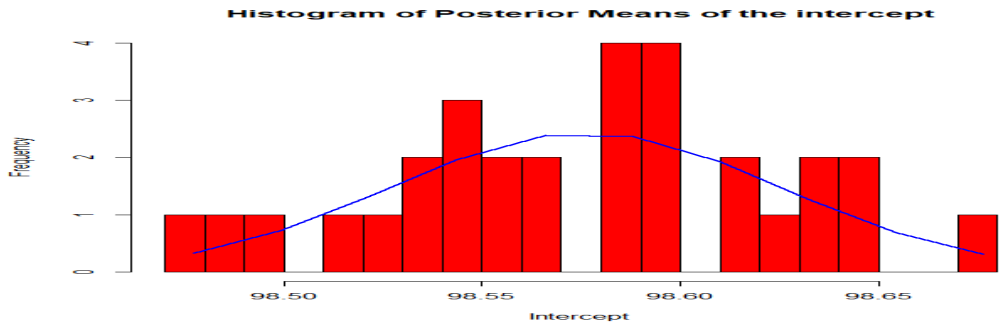

Fig. 6. Histogram with density curve for the intercept

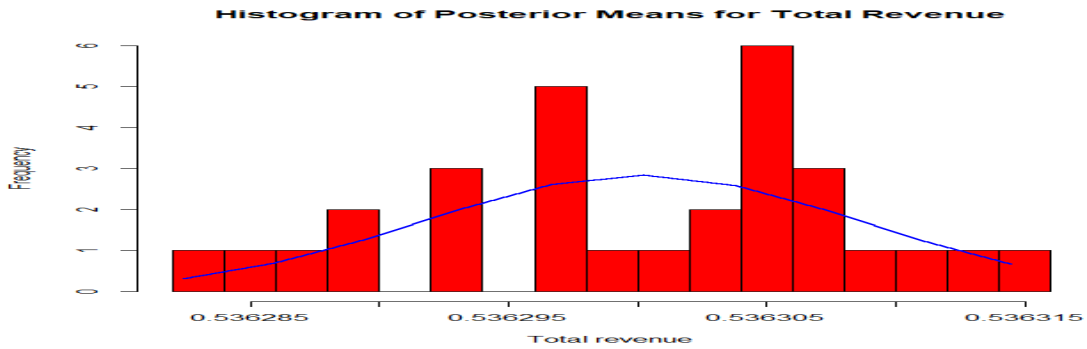

Fig. 7. Histogram with density curve for the Total Revenue

Table 3. Bayesian Regression for Normal-Gamma prior

	Post mean	Post Std.	2.5%	97.5%
Intercept	98.57242	0.06117094	-0.32344	0.1234
Total Revenue	0.5362977	0.00001096904	0.51232	0.77237
Precision	20.048005	1.428444	16.3245	22.1051

4.5 Comparison between the classical and the Bayesian results

Table 4 above shows the comparison between the ordinary least squares approach(OLS) and the Bayesian approaches used to fit the data. It could be seen that Bayesian estimate with Norma-gamma prior produces the better estimate for the Total Revenue as it produces estimate of 0.536 and posterior standard deviation of 0.00001.OLS estimates produces better next with estimate of 0.53630 and standard deviation 0f 0.05256.

Table 4. Classical versus Bayesian regression results

	OLS	Bayesian (normal-gamma)
Intercept	98.57128	98.57242
	(0.28307)	(0.06117094)
Total revenue	0.53630	0.5362977
	(0.05256)	(0.00001)
Precision		20.048005
		(1.428444)

The Posterior means and standard deviations in brackets

Forecast

Table 5 shows the forecast values for the Total expenditure using the Ordinary least squares Bayesian using uniform and Normal-gamma prior respectively. The forecast was done for the next 30 years. The forecast was made for years 2019 to 2048.

Table 5. Forecast values for total expenditure

Years	OLS	Normal-Gamma
2019	2971.024	2971.013
2020	3063.684	3063.672
2021	3156.344	3156.332
2022	3249.003	3248.991
2023	3341.663	3341.65
2024	3434.323	3434.31
2025	3526.983	3526.969
2026	3619.642	3619.628
2027	3712.302	3712.288
2028	3804.962	3804.947
2029	3897.622	3897.607
2030	3990.281	3990.266
2031	4082.941	4082.925
2032	4175.601	4175.585
2033	4268.261	4268.244
2034	4360.92	4360.903
2035	4453.58	4453.563
2036	4546.24	4546.222
2037	4638.9	4638.881
2038	4731.56	4731.541
2039	4824.219	4824.2
2040	4916.879	4916.86

Years	OLS	Normal-Gamma
2041	5009.539	5009.519
2042	5102.199	5102.178
2043	5194.858	5194.838
2044	5287.518	5287.497
2045	5380.178	5380.156
2046	5472.838	5472.816
2047	5565.497	5565.475
2048	5658.157	5658.135

5 Conclusion

Bayesian linear regression was used to fit a model between total expenditure on revenue in Nigeria for the periods, 1989 to 2018. The data fit normal distributionthus, wasused for the Likelihood. The constructed model fit was: Total Expenditure = 98.57128 + 0.53630* Total Revenue.This implied that a naira unit of the total expenditure will always be increased by 0.54of the total revenue. Averages of the expenditure spent and income/revenue received over the periord by the government were #2325.29 and #4152.02 respectively. Similarly, the total revenue explains 78% variations in the Total expenditure.

Bayesian estimate with elicited normal-gamma prior produceda better posterior meanof 0.536for the Total Revenuewith asmaller posterior standard deviation of 0.00001when compared with the OLS standard deviation of 0.05256. Forecast of 30 yearsfor the total expenditure using both OLS and Bayesian (normal gamma prior) were increasing as the yearswere progressing. Based on the conclusion, the following recommendations were made: Expenditure will increase every year so government should look for a way to increase revenue in order to be able to sustain the future expenses of the government; Government should see a way to control public expenditure through adequate budget implementation.

Competing Interests

Author has declared that no competing interests exist.

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