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Wavelet based Segmentation in Detecting Multiple Mean Changes in Time Series

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Short Research Article

Abstract

Aims/ Objectives: Multiple mean break detection problem in time series is considered. A segmentation based on detecting turning points is applied to the original time series and its scaling coefficients series resulting from the maximal overlapped discrete wavelet transform (MODWT). Using a segmentation level along with a minimal distance parameter between two successive turning points we select a small number of segments within each series. A change point statistical test is then run separately within each series and over each segment. The simulation experiment shows that the multiple mean break detection procedure offers very good practical performance. The test procedure is applied to a real set of data.

Keywords: Discrete wavelet transform; multiple mean break; segmentation; turning points; time series.

2010 Mathematics Subject Classification: 53C25, 83C05, 57N16.

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1 Introduction

Change point detection is a fundamental problem in the analysis of nonstationary time series. There is an extensive literature in identifying single break in the mean and other parameters such as the variance. Detection of several change-points are of interest in a wide range of applications such as stock prices and climatic data. In this paper we consider the problem of detecting simultaniously several mean breaks of a discrete time series by segmenting the series using the maximal overlapped discrete wavelet transform (MODWT) [1]-[3], and applying a segmentation procedure to both the original series and to the smoothed resulting scaling coefficients from the transform. Our approach is nonparametric and is quite different from the other related work such as the Baysian approach in [4] which suggest a recurcive algorithm to identify change-points using segmentation and assuming independence between segments, and [5] which follows the same Bayesian approach and make use of the advanced Monte Algorithm (SAMC). A nonparametric approach that was explored within the framework of least squares regression trees is given by [6]. We can mention as well the wild binary segmentation (WBS) proposed by [7] where the basic model is a deterministic piecewise-constant signal with multiple breaks plus a white noise. The method is efficient but make use of many technical assumptions. A good review and discussion of multiple breaks detection methods can be found in [8] which is an online repository of publications and sofware dedicated to break detections in data.

We assume that the break locations k_i^* and their numbers is unknown, but they are assumed to be relatively away from each other so that the sample size of any retained segment allows us to run a change-point statistical test designed for a single mean break detection. Several segmentation methods are available in the litterature, and in this work we explore the segmentation proposed in [9] which does not rely on any probabilistic assumption. The approach for obtaining such automatic segmentation is based on locating turning points which are defined first by identifying local maximum and minimum. In general these turning points indicate a change in the direction of local trends of the series. We emphasize here only the situation where a change in trends is occuring over different time periods. The basic idea of this procedure is to split the series into different periods such that each extracted segment is associated to a single trend. In order to gain the full benefit of such segmentation, we first apply it to the original series and then to their scaling coefficients up to some level J of the MODWT transform. The advantage of the MODWT transform is twofold, first we do not loose the existing different local trends in the original series and secondly the data is smoothed up to some level so that we nearly avoid the burden of how to choose the segmentation level. In our simulation we show that a good practical choice of the segmentation level is to start with a higher level for the serie X_t and then lower that level as we move from lower to higher scales for the scaling coefficient of the MODWT.

Its important to note here that this procedure does not require to run the wavelet transform to a higher level, the segmentation works better for lower scale of the MODWT transform as can be seen in Fig. 2.

The proposed procedure begin first by segmenting the series X_t and its scaling coefficients by identifying separately the existing turning points and counting their numbers within each series. Then based on these points, as described in the algorithm in section 3 a new segmentation is derived where each turning point is within or in the middle of these new segments. The corresponding time interval of these new segments obtained from each series are then used to segment both the series and its scaling coefficients. Each turning point within each series is then subject to a statistical test and checked whether its a true change in the mean.

The outline of this paper is as follows. Section 2 provides a brief review of the MODWT tranform in the literature. The scaling coefficients for a series with multiple mean breaks are depicted in this section. Section 3 introduces how turning points are selected along with a segmentation algorithm. In section 4 we give a summary of one of the mean statistical test used in a single change point detection problem. Section 5 presents some Monte Carlo experiments. In section 6 we apply the test procedure to a real data, a Well-Log series. Section 7 provides a some conclusions.

Let $\mathbf{X} = (X_1, \cdots, X_N)$ be an observed time serie for which we want to test for several breaks in the mean. Assume that $\mathbf{X}$ is regarded under H_0 as a realization of discrete time stochastic process with constant mean, and constant variance.

The null hypothesis of our test problem would be

$$H_0 : E(X_t) = \mu \quad t = 1, \cdots, N \tag{1.1}$$

which we whish to test against the alternative hypothesis

$$H_1 : E(X_t) = \mu_i \quad \text{for} \quad k_{i-1} \leq t < k_i, \ i = 1, .., K << N \tag{1.2}$$

where $k_0 = 1$ and the times $t = k_i$ of mean change are unknown.

2 The MODWT Transform

The maximal-overlap discrete wavelet transform (MODWT) also called the undecimated or shift invariant discret wavelet transform has been discussed in the wavelet literature see [1]-[3]. For the class of discrete compactly supported Daubechies wavelets, we denote $\{\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}, \quad l = 0, \cdots, L_j - 1\}$ the level j of a wavelet and scaling filters of length $L_j = (2^j - 1)(L - 1) + 1$. These filters are discussed and given in more details in [2]. The stochastic processes resulting from applying these filters to $\{X_t\}$ are respectively given by the level j wavelet and scaling coefficients

$$V_{j,t} = \sum_{l=0}^{L_j - 1} \tilde{g}_{j,l} X_{t-l} \qquad j = 1, 2 \cdots J \tag{2.1}$$

The MODWT wavelet and scaling coefficient based on a finite sample size are given by

$$\text{and } \ \tilde{V}_{j,t} = \sum_{l=0}^{L_j - 1} \tilde{g}_{j,l} X_{t-l \ mod \ N}, \ t = 0, 1, \cdots N - 1 \tag{2.2}$$

obtained as a result of circularly filtering $X_0, X_1, \cdots, X_{N-1}$ with the filters $\{\tilde{h}_{j,l}\}$ and $\{\tilde{g}_{j,l}\}$. Where $X_{t \ mod \ N} = X_t$ if $t \geq 0$ and $X_{t \ mod \ N} = Y_{N-|t|}$ if $t < 0$. Note that $\tilde{V}_{j,t} = V_{j,t}$ for $t \geq L_j - 1$ and $j = 1, \cdots, J$ where J is the maximum level up to which we run the MODWT.

Its important to note that in general the wavelet coefficients remove trends and demean the process X_t as shown in [10], whereas the scaling coefficients $\tilde{V}_{j,t}$ preserve the overall shape of X_t and keep track of all important data points, and particularly the turning points that detect direction change in the local trends. These changes should be regarded as a change in the mean of the original series X_t as shown Fig. 1.

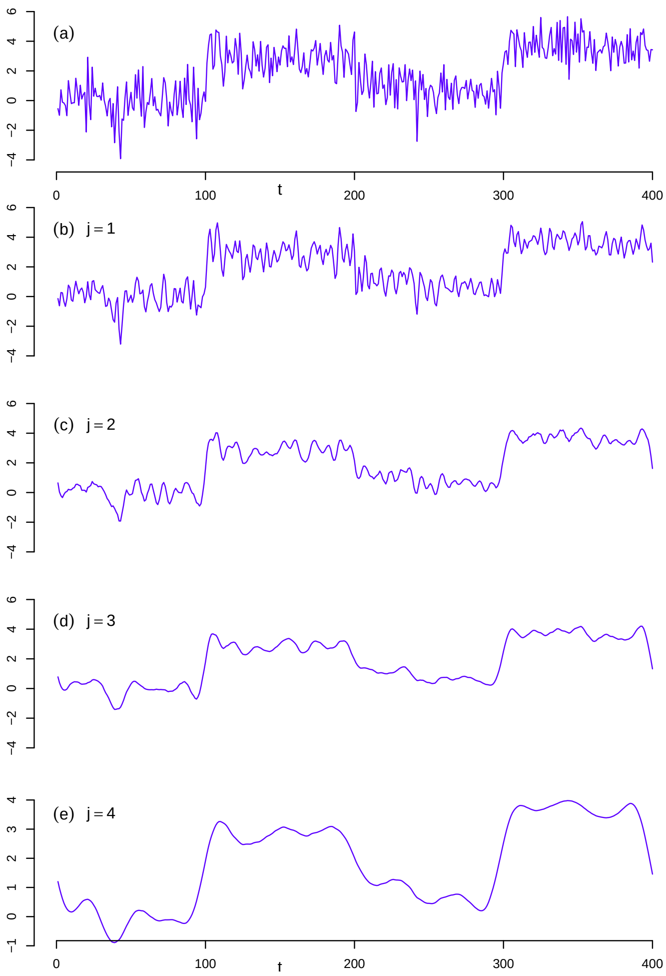

Fig. 1. (a)Gaussian G_t serie with multiple breaks at $k_1 = 100$, $k_2 = 200$ and $k_3 = 300$. (b)-(e) are the scaling coefficients $\tilde{V}_{j,t}$ respective for level $j = 1, 2, 3, 4$. The R wavelet package was used to compute the MODWT with time aligned

3 Segmentation Procedure

Turning points are defined as local minimum or maximum points in a time series, they are identified as the start and end of a local trend. They are used to segment the series into different periods such that each extracted segment is expected to cover a single trend over that period. In this work we explore the segmentation procedure described by the Algorithm 2 in [9]. The proposed method generates segments at different levels of details up to a specified segmentation level S_L, and does not depend on any threshold. A set of turning points is selected at a specified level of the segmentation. This segmentation level aim to reduce the number of tuning points by combining small trends into large ones. The series X_t and the scaling coefficients $\tilde{V}_{j,t}$ $j = 1, \cdots, J$ are then segmented separately according to the previous method. It should be noted here that because the scaling coefficients are smoother than X_t, different lower segmentation level are used for each $\tilde{V}_{j,t}$.

A small number of turning points are then identified within each series. These are regarded as possible change points to be tested. To this end we use these set of identified turning points to split separately the series X_t and $\tilde{V}_{j,t}$ $j = 1, \cdots, J$ into new set of segments such that for each series each point of the selected set of turning point belongs to at least one of these segments. Obviously for a given series this procedure allows for a situation where one turning point might belong to more than one segment. This might occurs particularly if some of these identified points are too close, and in this case in order to maintain the assumption that two successive breaks are away from each other, we need to set a minimal distance between two successive turning points. For that purpose we introduce a parameter λ_d that specifies the minimal distance we should have between two successive turning points. This help to further reduce the number of these points. A practical choice of λ_d would depend on the segment size N_s. Our simulations show that for the particular case $N_s = 100$, we should set it so that $\lambda_d \geq 6$. As a general rule, based on simulation experements and the example of real data, we recommend any value around 10% of the segment size N_s. Note that setting λ_d does not guarantee any two segment to be disjoints. When a turning point is within two different segments that are not disjoints, then it will be subject twice to the same statistical test but with different data. This should only strengthen the chance of good detection. As illustrated in the example of simulated models with three mean breaks, the number of turning points was in general less than 20 for a sample size of $N = 400$, and the correct detection of the true breaks were very high for the Gaussian model G_t and its square G_t^2 as shown in the table 1. The next algorithm describes the main steps of the new segmentation to split the series X_t and the scaling coefficients $\tilde{V}_{j,t}$.

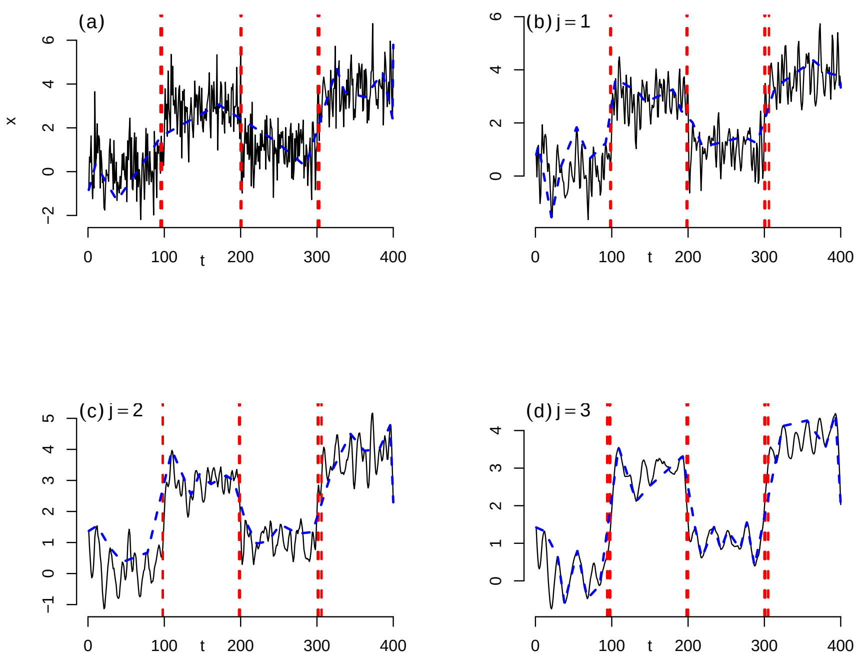

Fig. 2. (a) Gaussian serie G_t with multiple mean breaks. (b)-(d) are the scaling coefficients $\tilde{V}_{j,t}$ respective for level $j = 1, 2, 3$. The segmentation of each series is plotted in dashed lines. The vertical lines are the detected breaks after running the statistical test over the set of segments within each series

3.1 Algorithm

Let $\{X_t, t = 1, \cdots, N\}$ be an observed time series and $\{p_k, k = 1, \cdots, K\}$ be the set of turning points locations ordered in times retained from its segmentation with $K > 2$ and a segmentation level S_L according to the procedure in Algorithm 2 of [9]. Denote by N_s the minimum segment size to be used so that the power of the statistical test is not affected. An arbitrary choice of the minimum distance parameter λ_d is chosen accordingly, and if two successive turning points are too close we discard the smallest in magnitude. Then given the random location of these turning points, one way to split up X_t into segments $\mathbf{Y}_k$ is as follow

Algorithm

(1) for $k = 1$
    if($p_1 \geq N_s/2$)
        $\mathbf{Y}_1 = X[1 : (p_1 + N_s/2)]$
    else $\mathbf{Y}_1 = X[1 : (p_1 + N_s]$

(2) set $k = 2$
while ($k \leq K$)
    if ($p_k \geq p_{k-1} + N_s/2$) and ($p_k < N - N_s/2$)
        $\mathbf{Y}_k = X[(p_{k-1} + 1) : (p_k + N_s/2)]$
    else $\mathbf{Y}_k = NULL$
    $k = k + 1$.

Note that when a turning point is near the end of the series, and if we cannot form a segment of size at leat N_s, then we set $\mathbf{Y}_k = NULL$ and consider the previous segment as the last one. This shows that in order to increase the chance of getting a segment for the end of the series we should choose smaller N_s.

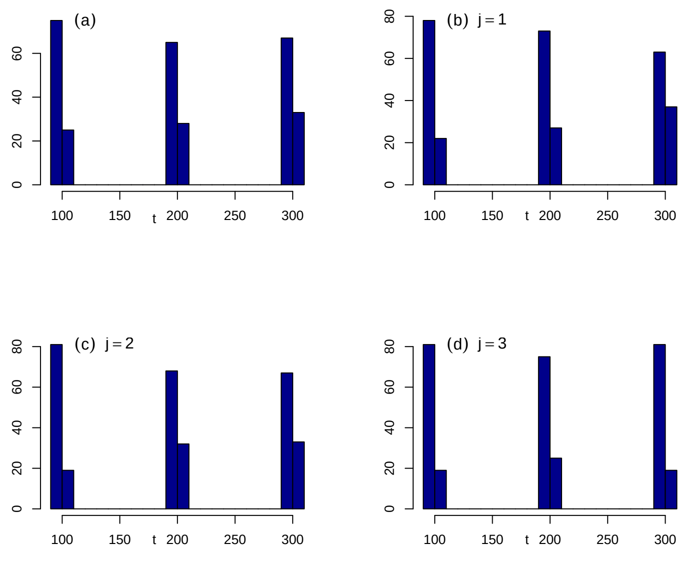

Fig. 3. Estimate of the break locations k_i^* which correspond to the highest value of the test statistic. (a) Gaussian serie G_t, (b)-(d) scaling coefficients $\tilde{V}_{j,t}$ of G_t respective for level $j = 1, 2, 3$

The same segmentation procedure is applied also to the scaling coefficients $\tilde{V}_{j,t}$ for $j = 1, \cdots, J$ where X_t is replaced with $\tilde{V}_{j,t}$. For a fixed series, based on the locations of the set of selected turning points a new segmentation is derived by the previous Algorithm such that each turning point is within a segment of size at least equal N_s.

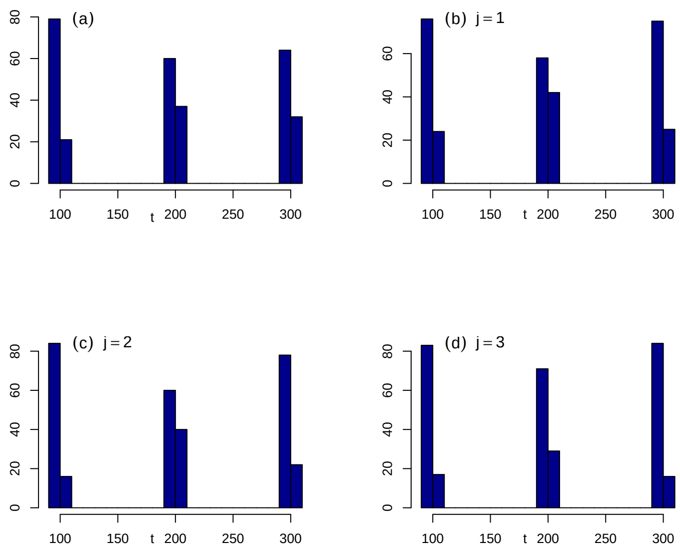

Fig. 4. Estimate of the break locations k_i^* for the square of the Gaussian serie and its scaling coefficients as in Fig. 3.

4 Change Point Statistic

The constructed segments $\mathbf{Y}_k, k = 1, \cdots, K$ are assumed to contain all mean break points and are then subject to a statistical test. Due to the random fluctuations in the data that affect both segmentations, the number of retained segments $\mathbf{Y}_k$ according to the previous algorithm should be higher than the true number of breaking points. This only increases the probability of covering all the mean break points by these segments. Note that each turning point within a segments $\mathbf{Y}_k$ is subject to a statistical test to check whether its in fact a true change point. There are many statistical tests available in the literature that can be applied for this purpose. We choose to apply a nonparametric test proposed by [11] and [12]. The basic idea of this test suggests to bypass the estimation problem of the long run variance as discussed in [13] and [14] by applying a self normalizer to the test statistic. Two types of self normalisation were introduced in this context, the first is based on the proposed test statistic in [11] and the second type is proposed in [12]. We make use of the second type, and in our setting, we define the partial sums computed from the scaling coefficients $\tilde{V}_{j,t}$ for $j = 1, \cdots, J$.

$$S_j(t_1, t_2) = \sum_{t=t_1}^{t_2} \tilde{V}_{j,t}, \text{ if } L_j - 1 \le t_1 < t_2 \text{ and } 0 \text{ otherwise}$$

Then the normalization process is defined for $k = L_j - 1, L_j, \cdots, N-1$ by

$$R_j(k) = \frac{1}{N_j^2}\left[\sum_{t=L_j-1}^{k}(S_j(1,t) - \frac{t}{k}S_j(1,k))^2 + \sum_{t=k+1}^{N-1}(S_j(t,N) - \frac{N-t+1}{N-k}S_j(k+1,N))^2\right] \quad (4.1)$$

The statistic is then given by

$$Q_{V_j} = \max_{L_j-1\leq k\leq N-1}\frac{T_j^2(k)}{R_j(k)} \quad (4.2)$$

where $T_j(k) = \frac{1}{\sqrt{N_j}}\sum_{t=L_j-1}^{k}(\hat{V}_{j,t} - \overline{V_j})\ \ k = L_j - 1, \cdots, N$

A similar statistic Q_X based on X_t is computed where $\tilde{V}_{j,t}$ is replaced with X_t.

The asymptotic distribution of Q_{V_j} or Q_X is not standard, its critical values are computed by means of simulations, and are tabulated in [12]. Based on the scaling coefficients $\tilde{V}_{j,t_i}$, the test statistic Q_{V_j} is computed seperatly over each segment $\mathbf{Y}_k$ and compared to its critical values. For instance, for $\alpha = 5\%$, then the simulated 95% quantile value is 40.1.

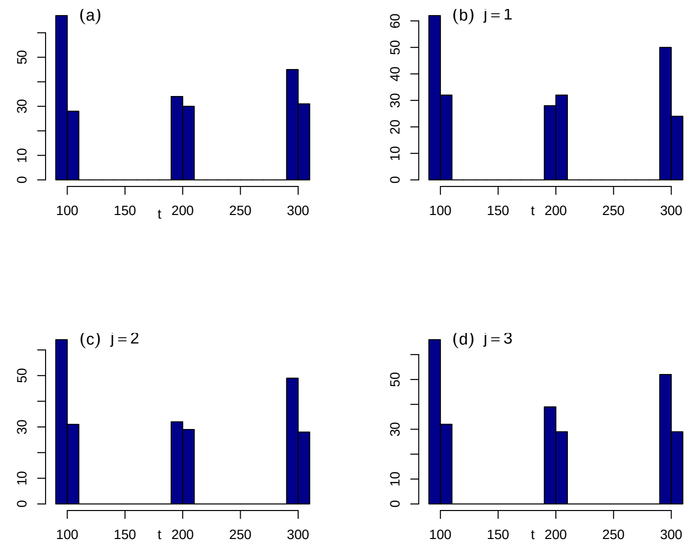

Fig. 5. Estimate of the break locations k_i^* for the $AR(1)$ serie and its scaling coefficients as in Fig. 3

Any turning point that is selected by this statistical test as a breaking point is retained. If the same point is selected more than one time because its belong to two different non disjoint segments, and if the estimated locations in time are different, then we can adopt a rule to choose the point location that correspond to the highest value of test statistic Q_X or Q_{V_j}.

5 Simulations

To illustrate the proposed procedure, we considere three different simulated models each with sample size $N = 400$ given in the example below. We set $\mu_1 = 0$, $\mu_2 = 3$, $\mu_3 = 1.5$ and $\mu_4 = 3.5$ so that each series start with $\mu_1 = 0$, and then we have three breaks in the mean at times $k_1 = 100$, $k_2 = 200$ and $k_3 = 300$. The process $\{\epsilon_t\}$ is a Gaussian uncorrelated sequence with mean zero and variance

1. The Least Asymetric wavelet filter $LA(8)$ of length $L = 8$ was used to run the transform up to level $J = 3$. The choice of the wavelet filter was arbitrary with moderate length. In order to get the set of turning points we apply the segmentation procedure in [9] with segmentation level $S_L = 4, 3, 2$ and 1 respectively for X_t and $\tilde{V}_{j,t},\ j = 1, 2, 3$, then by using the proposed algorithm in section 3 with $N_s = 100$ and $\lambda_d = 10$ we derive a new set of segments $\mathbf{Y}_k$ for $k = 1, \cdots K$ separately for the original series X_t and $\tilde{V}_{j,t},\ \ j = 1, 2, 3$. We then run the statistical test given in (4.2) over each segment $\mathbf{Y}_k$. Based on the test result we decide whether there exits a change point in a given segment. Any break detection is then retained and plotted as shown in Fig. 2 for the Gaussian model.

This process of generating data sets, running the above procedure separately for each series and then testing at significance level $1-\alpha = 95\%$ over each segment $\mathbf{Y}_k$ was repeated a total of $n = 100$ times for each sample size. The proportion of true break detection for these replications in summarized in Table 1 for three models. For illustration, the results for a single replication for model (a) are shown in Fig. 2.

5.1 Example

We consider two linear models G_t (Gaussian) and $AR(1)$, and a nonlinear model G_t^2 each with three breaks in the mean occuring respectively at times $k_1 = 100$, $k_2 = 200$ and $k_3 = 300$.

(a) Gaussian model

$$X_t = \mu_1 I_{(0\le t<100)} + \mu_2 I_{(100\le t<200)} + \mu_3 I_{(200\le t<300)} + \mu_4 I_{(300\le t\le 400)} + \epsilon_t \tag{5.1}$$

(b) Non-linear model

$$X_t = \mu_1 I_{(0\le t<100)} + \mu_2 I_{(100\le t<200)} + \mu_3 I_{(200\le t<300)} + \mu_4 I_{(300\le t\le 400)} + \epsilon_t^2 \tag{5.2}$$

(c) $AR(1)$ model

$$X_t = 0.7X_{t-1} + \mu_1 I_{(0\le t<100)} + \mu_2 I_{(100\le t<200)} + \mu_3 I_{(200\le t<300)} + \mu_4 I_{(300\le t\le 400)} + \epsilon_t \tag{5.3}$$

6 Well Log Data

In order to illustrate the performance of the proposed procedure for real data, we consider the example of well log time series data shown in Fig. 6. The data is available from the website:

http://mldata.org/repository/data/viewslug/well-log/ This well log data set is for detecting changes in the rocks stratification, described in [15]. It consists of 4050 nuclear magnetic resonance measurement taken from drill while drilling a well. A plot of the data shows that the series contains large spikes, so we only consider the middle portion X_t with no outliers of size $N = 1100$ which shows that there exist at least 7 breaks in the mean. Our concern is to put into practice the performance of the above test procedure. The plots in figure 6 clearly shows detection of multiple mean breaks as indicated by the vertical dashed lines. The segment size was set to $N_s = 150$, the minimal distance $\lambda_d = 20$, and we applied different segmentation levels for each series. All the 7 visible breaks are successfully detected in all panels, and there are extra breaks detected in panel (a) for X_t and in panel (d) for $\tilde{V}_{3,t}$. The results for the scaling coefficients $\tilde{V}_{1,t}$ and $\tilde{V}_{2,t}$ in panels (b) and (c) are much better than in (a) and (c). For each series, all the estimated break locations are given in Fig. 7. We expect that the statistical test procedure might provide more than one closer location estimate for each break as shown in Fig. 7 where each panel is an histogram that shows how many times each single break is detected as a mean break. For instance the break that occurs just after time $t = 400$ was detected 6 times in X_t, 4 times in $\tilde{V}_{1,t}$, 6 times in $\tilde{V}_{2,t}$ and 8 times in $\tilde{V}_{3,t}$.

Table 1. Proportion in percentage (%) of true mean break detection from $n = 100$ replications each with sample size $N = 400$. The true time location of breaks are k_1, k_2 and k_3

Model	$k_1 = 100$	$k_2 = 200$	$k_3 = 300$
G_t	100	93	100
$V_{1,t}$	100	100	100
$V_{2,t}$	100	100	100
$V_{3,t}$	100	100	100
G_t^2	100	97	96
$V_{1,t}$	100	100	100
$V_{2,t}$	100	100	100
$V_{3,t}$	100	100	100
$AR(1)$	95	64	76
$V_{1,t}$	94	60	74
$V_{2,t}$	95	61	77
$V_{3,t}$	98	68	81

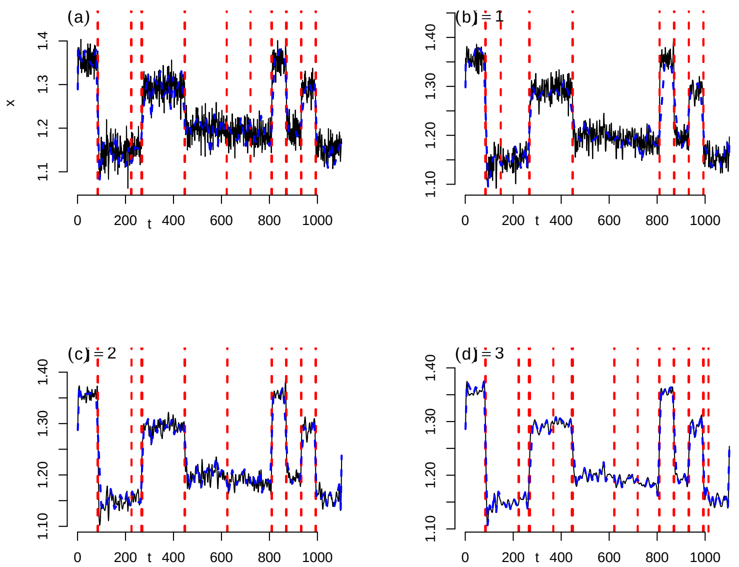

Fig. 6. The segmentation of the Well Log data in dashed line with segmentation level $4, 3, 2, 1$ respectively for data (a) X_t , and $\tilde{V}_{j,t}$, $j = 1, 2, 3$ (b)-(d). The vertical dashed lines are the detected breaks

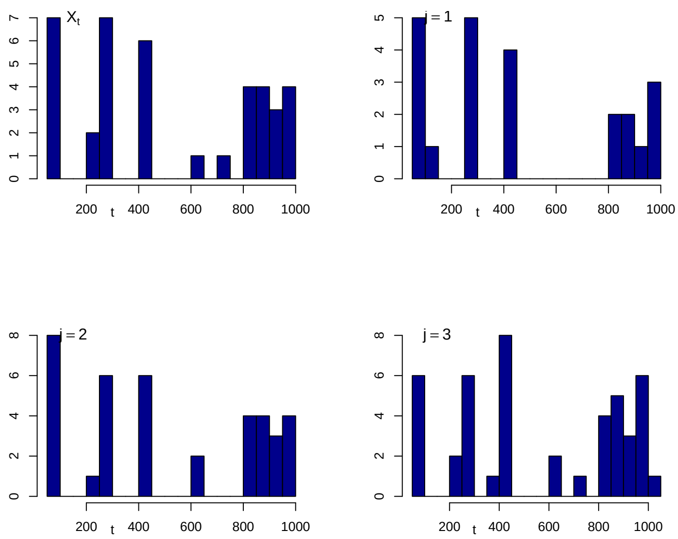

Fig. 7. Estimate of the break locations in (a) X_t, and $\tilde{V}_{j,t},\ j = 1, 2, 3$ (b)-(d)

7 Conclusion

The problem of detecting multiple mean changes in time series seems to be a difficult issue to solve particularly if we adopt the nonparametric approach without any assumption on the time series model. The wavelet transform comes as a handy very useful mathematical tool that can provide a smoother representation of the series X_t without loosing details in the shape of the existing local trends. In fact as can be seen from Fig. 2, the scaling coefficients $\tilde{V}_{j,t}$ offers to run the same statistical test using differents data set for the same test problem. In order to search and locate the mean breaks, a segmentation procedure is used to subdevise in an appropriete way the sample time interval of each series so that each break belongs to at least one segment. Because we do not have a prior information about the number of breaks, we could end up with a large number of turning points if we rely on segmentation alone from the Algorithm in [9]. The minimum distance parameter λ_d was introduced to overcome this problem and therefore reduces the number of turning points which helps in getting a small number of segments. As illustrated in Table 1, the proportion of true mean breaks detection is very high and around 100% for the gaussian model G_t and model G_t^2, and the detection proportion by the scaling coefficients are in general higher than in the original seies. This proportion get slighly lower for the $AR(1)$ model, and particularly for the second break. It should be noted as well that the power of the statistical test used here may be affected due to the varying small sample size of a segment $\mathbf{Y}_k$. In general the overall performances of this test procedure are good enough, and can be improved by allowing moderate sample size for $\mathbf{Y}_k$, which is arbitrary chosen in our procedure. The approach used by the segmentation requires the setting of the segmentation level S_L and the minimal distance λ_d between successive turning points. These are left to be set by the user according to the shape of the data at hand. The tracking of the locals up trends and down trends when we move from lower to higher scales in the MODWT can be improved by trying different values for the parameters S_L and λ_d. It should be noted here as well that the choice of the wavelet filter was also arbitrary and any other wavelet filter could well be applied.

Competing Interests

Author has declared that no competing interests exist.

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