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Identification of Heteroscedasticity in the Presence of Outliers in Discrete-Time Series

Emmanuel Alphonsus Akpan[1*], K. E. Lasisi[1] and Ali Adamu[2]

[1]Department of Mathematical Science, Abubakar Tafawa Balewa University, Bauchi, Nigeria.
[2]Federal College of Education (Tech.), Gombe, Gombe State, Nigeria.

Authors' contributions

This work was carried out in collaboration between all authors. Author EAA designed the study, performed the statistical analysis, wrote the protocol, wrote the first draft of the manuscript and managed the literature searches. Authors KEL and AA managed the analyses of the study. All authors read and approved the final manuscript.

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Abstract

This study considered the effects of outliers on the identification of heteroscedasticity in the daily closing share price returns series of Diamond Bank, Fidelity Bank and Skye bank using correlogram, Ljung-Box test and Lagrange Multiplier test. The data were obtained from Nigerian Stock Exchange from January 3, 2006, to November 24, 2016, and comprises 2690 observations. About Seventeen outliers were detected in the return series of Diamond bank, sixteen outliers identified in the return series of Fidelity bank and twenty-six outliers found in Skye bank, and their effects were removed to achieve an outlier adjusted series for respective banks under study. Meanwhile, heteroscedasticity was found to exist in the two (the outlier contaminated and the outlier-adjusted) series. However, the results of our findings indicated that outliers could hide significant heteroscedasticity in correlogram, minimize the power of Ljung-Box test and amplify the power of Lagrange Multiplier test. The implication is that failure to account for outliers would result in impaired or spurious heteroscedasticity detection in discrete-time series. Thus, the strength of this study is in highlighting the undesirable effects of outliers on heteroscedasticity detection.

**Corresponding author: E-mail: eubong44@gmail.com;*

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1 Introduction

Heteroscedasticity means changing variance. It is a phenomenon that occurs when the assumption of constant variance is violated. The existence of heteroscedasticity commonly called the ARCH effect is a very common occurrence in time series data especially financial time series data. A major setback to linear stationary models when applying to financial data (returns series) is their failure to account for changing variance. Neglecting the presence of heteroscedasticity in linear models results in inefficient ordinary least squares estimates of ARIMA parameters though still consistent and asymptotically normally distributed, their variance-covariance matrix is no longer the usual one. Thus, making the t-statistics invalid and cannot be used to examine the significance of the individual explanatory variables in the model [1,2]. Also, over-parameterisation of an ARIMA model and low statistical power are identified as part of the consequences for neglecting heteroscedasticity. In addition, neglecting heteroscedasticity can lead to spurious nonlinearity in the conditional mean and difficulty in computing the confidence interval for forecasts [3,4,1,2].

On the other hand, another very common attribute in time series data is the presence of outliers. Outliers in homoscedastic model make the model heteroscedastic, distorting the diagnostic tools for heteroscedasticity such that it may not be correctly identified. Similarly, [5] further affirmed and maintained that outliers affect the identification of conditional heteroscedasticity and the estimation of GARCH models. Also, it is evident according to [6] that outliers have a great impact on the existing heteroscedasticity tests and the estimators of the heteroscedastic models. Such impact of outliers on the diagnostic tools for heteroscedasticity is well defined in [7]. They showed that both the asymptotic size and power properties of Lagrange Multiplier (LM) test for ARCH/GARCH are adversely affected by outliers, particularly, additive outliers. Furthermore, [8] found that the order of identification, t-statistics and corresponding p-values of the estimates of GARCH parameters are affected by outliers in an unexpected manner. Therefore, it could be argued that it is gainful to take into consideration the presence of outliers whenever heteroscedasticity is modeled.

The fact that previous studies in Nigeria have failed to consider the presence of outliers while modelling heteroscedasticity in stock returns has provided a novel ground for this study. For instance [9] investigated the time series behaviours of daily stock returns of four firms listed in the Nigerian Stock Market from January 2, 2002 to December 31, 2006 using three different models of heteroscedastic process, namely; GARCH(1,1), EGARCH(1, 1) and GJR-GARCH(1, 1) models, respectively. The four firms whose share prices were used in the analysis were United Bank for Africa, Unilever, Guinness and Mobil. All return series exhibit leverage effect, leptokurtosis, volatility clustering and negative skewness which are common to most economic financial time series. The estimated results revealed that the GJR-GARCH (1, 1) gives a better fit to the data and are found to be superior both in-sample and out-sample forecasts evaluation.

[10] examined the response of volatility to negative and positive news using daily closing prices of the Nigerian Stock Exchange (NSE). By applying EGARCH (1, 1) and GJR-GARCH (1, 1) models to NSE daily stock return series from January 2, 1996 to December 30, 2011. They found strong evidence supporting asymmetric effects in the NSE stock returns but with the absence of leverage effect. Specifically, the estimates from EGARCH model showed positive and significant asymmetric volatility coefficient. In the same way, results of the GJR-GARCH showed negative and significant asymmetric volatility coefficient, also, supporting the existence of positive asymmetric volatility. Overall results from this study provided support for positive news producing higher volatility in the immediate future than negative news of the same magnitude in Nigeria.

[11] studied the modeling and forecasting of daily returns volatility of Nigerian Banks Stocks using data from January 4, 2005 to August 31, 2012. Three symmetric models ARCH (1), ARCH (2) and GARCH (1, 1) and two asymmetric models EGARCH (1, 1) and TARCH (1, 1) were used in capturing the volatility pattern of the banks stocks. The findings of the study revealed that the return series were stationary but not normally distributed with the presence of ARCH effect. Furthermore, the results of post-estimation

evaluation revealed that asymmetric conditional heteroscedastic models are more suitable for modeling daily returns volatility of Nigerian Banks stocks compared with symmetric heteroscedastic models.

[12] looked at a possible combination of both ARMA and ARCH-type models to form a single model such as ARMA-ARCH that will completely model the linear and non-linear features of financial data. Daily closing share prices of First Bank of Nigeria plc from January 4, 2000 to December 31, 2013 were considered. The study provided evidence to show that ARMA (2, 2) model was adequate in modeling the linear dependence in the returns while ARCH (1) model was adequate in modeling the changing conditional variance in the returns. Hence, ARMA (2, 2)-ARCH (1) model completely modeled the returns series of First Bank of Nigeria.

[13] detected and modeled the asymmetric GARCH effects in a discrete-time series by exploring the share price returns of Zenith bank plc obtained from the Nigerian Stock Exchange from January 4, 2006 to May 26, 2015. The study applied sign and size test to identify the asymmetric GARCH effects and modeled by EGARCH and TGARCH respectively with respect to normal distribution. The findings of the study revealed that the asymmetric effect was adequately captured modeled by EGARCH (0, 1) and TGARCH (0, 1) models. Yet they did not take into account the presence of outliers.

Specifically, the aim of this study is to identify the effects of outliers on the tools (correlogram, Ljung-Box test and Lagrange Multiplier test) used for heteroscedasticity detection. Moreover, the remaining part of this work is organized as follows; section 2 handles the methodology to be explored then followed by analysis and discussion of results in section 3 while the conclusion of overall results is treated in section 4.

2 Materials and Methods

2.1 Return

The return series R_t can be obtained given that P_t is the price of a unit share at time, t and P_{t-1} is the share price at time t−1.

$$R_t = \nabla lnP_t = (1-B)lnP_t = ln\, P_t - ln\, P_{t-1}. \quad (1)$$

The R_t in equation (1) is regarded as a transformed series of the share price, P_t meant to attain stationarity, that is, both mean and variance of the series are stable [14]. The letter $\boldsymbol{B}$is the backshift operator.

2.2 Autoregressive Integrated Moving Average (ARIMA) model

[15] considered the extension of ARMA model to deal with homogenous non-stationary time series in which X_t, itself is non-stationary but its d^{th} difference is a stationary ARMA model. Denoting the d^{th} difference of X_t by

$$\varphi(B) = \phi(B)\nabla^d X_t = \theta(B)\varepsilon_{t,} \quad (2)$$

where$\varphi(B)$ is the nonstationary autoregressive operator such that d of the roots of $\varphi(B) = 0$ are unity and the remainder lie outside the unit circle. $\phi(B)$ is a stationary autoregressive operator.

2.3 Tools for identification of heteroscedasticity

Correlogram: If at least one lag term in both ACF and PACF of squared residual series is found to be statistically significant, then the presence of ARCH effect is confirmed [13,16].

Ljung - Box Test is given as

$$Q(m) = T(T+2) \sum_{l=1}^{m} \frac{\hat{\rho}_l^2 a_t^2}{T-l}, \tag{3}$$

where T is the sample size, m is a properly chosen number of autocorrelations used in the test, $\hat{\rho}_l^2(a_t^2)$is the lag- l ACF of a_t^2 [17]. If the entertained linear model is adequate, Q(m) is asymptotically a Chi-squared random variable with m – p – q degrees of freedom [18].

Lagrange Multiplier Test: Another approach for testing the ARCH/GARCH effect (otherwise called heteroscedasticity is the changing conditional variance) is to apply the Lagrange Multiplier (LM) test of ARCH(q) against the hypothesis of no ARCH effects to $\{a_t^2\}$ series. The LM test is carried out by computing, $\chi^2 = TR^2$ in the regression of a_t^2 on a constant and q lagged values. T is the sample size and R^2 is the coefficient of determination. Under the null hypothesis of no ARCH effects, the statistic has a Chi-square distribution with q degrees of freedom. If the LM test statistic is larger than the critical value, then, there is evidence of the presence of ARCH effect [19].

Outliers in Time Series: An outlier is an observation that diverges from an overall pattern on a sample. Generally, a time series might contain several, say k outliers of different types and we have the following general outlier model;

$$Y_t = \sum_{j=1}^{k} \omega_j V_j(B) I_t^{(T)} + X_t, \tag{4}$$

where $X_t = (\theta(B)) / (\varphi(B))\, a_t$, $V_j(B) = 1$ for an AO, and $V_j(B) = \frac{\theta(B)}{\varphi(B)}$ for an IO at $t = T_j$, $V_j(B) = (1 - B)^{-1}$ for a LS and $V_j(B) = (1 - \delta B)^{-1}$ for a TC. For more details on the types of outliers and estimation of the outliers effects see [20,21,15,22,23,24].

Moreover, in financial time series, the residual series, a_t is assumed to be uncorrelated with its own past, so additive, innovative, temporary change and level shift outliers coincide, and where both the mean and variance equations evolves together, we have

$$R_t - \mu_t = \tilde{a}_t + \omega I_t^{(T)}, \tag{5}$$

$$\tilde{a}_t = \sigma_t e_t, \tag{6}$$

$$\sigma_t^2 = \alpha_o + \alpha_1 \tilde{a}_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{7}$$

where $\tilde{a}_t$ is the outliers contaminated residuals.

3 Results and Discussion

This study considers the daily closing share prices of three major banks in Nigeria; Diamond bank, Fidelity bank and Skye bank and were obtained from the Nigerian Stock Exchange through the data range from January 3, 2006 to November 24, 2016 and comprises 2690 observations.

3.1 Time Series Plot Interpretation

Figs. 1 - 3 represent the share price series for the three banks. It could be observed that the share prices of all the banks do not fluctuate around a common mean. Thus clearly indicate the presence of a stochastic trend in the share prices, implying non-stationarity.

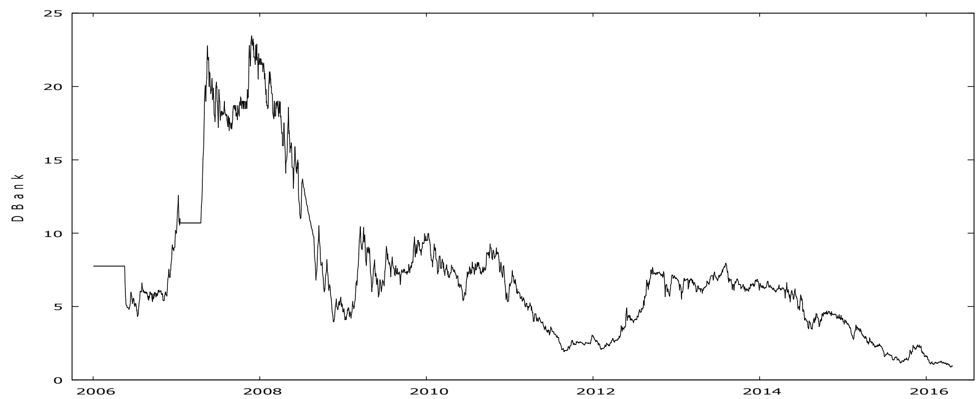

Fig. 1. Share price series of diamond bank

Fig. 2. Share price series of fidelity bank

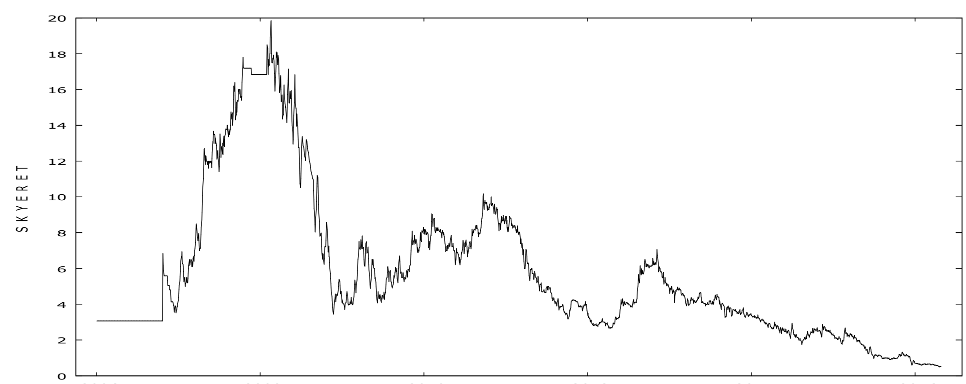

Fig. 3. Share price series of Skye bank

Since the share price series is found to be non-stationary, the first difference of the natural logarithm of share price series is taken to obtain a stationary (returns) series. The inclusion of the log transformation is to stabilize the variance. Figs. 4-6 show that the returns series appear to be stationary and they suggest that volatility clustering is quite evident in the different series.

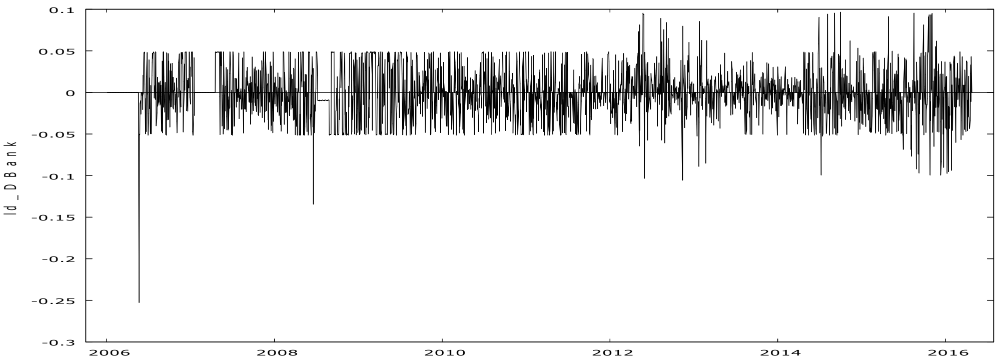

Fig. 4. Return series of diamond bank

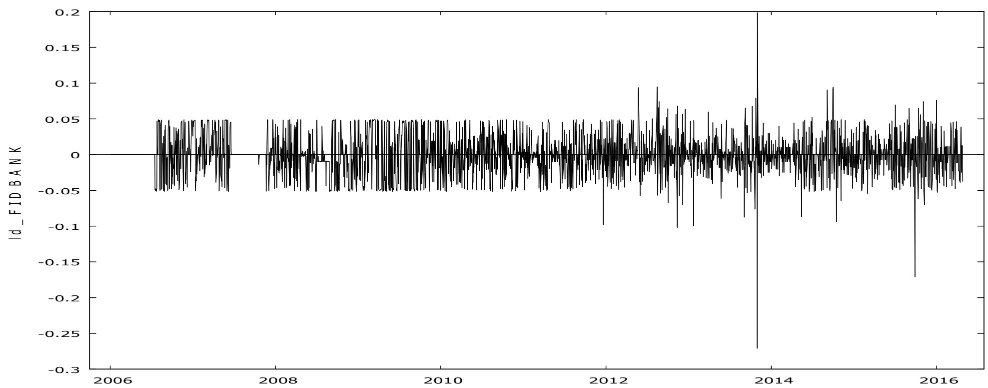

Fig. 5. Return series of fidelity bank

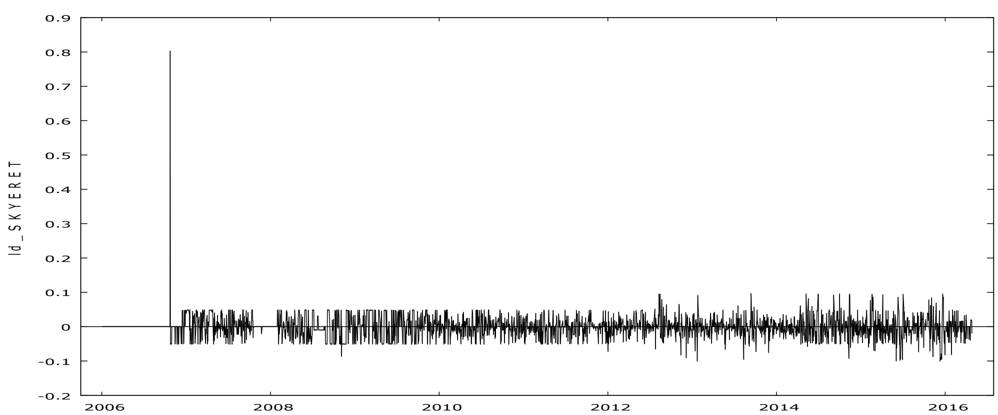

Fig. 6. Return series of Skye bank

3.2 Diamond Bank

From Figs. 7 and 8, both ACF and PACF indicate that mixed model could be entertained. The following models, ARIMA (1, 1, 1), ARIMA (1, 1, 2) and ARIMA (2, 1, 1) are entertained tentatively.

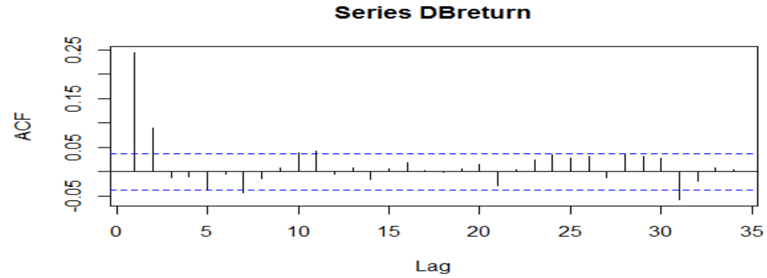

Fig. 7. ACF of return series of diamond bank

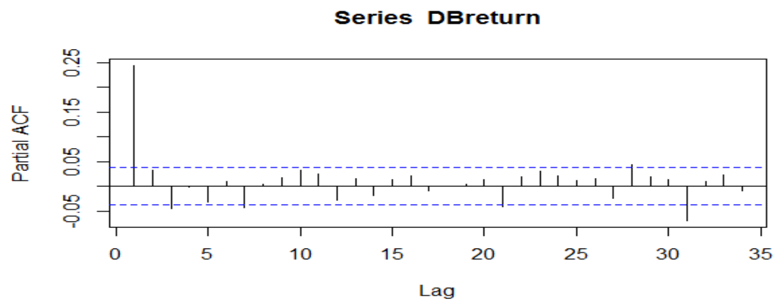

Fig. 8. PACF of return series of diamond bank

From Table 1, ARIMA (2, 1, 1) model is selected based on the ground of significance of the parameters and minimum AIC.

Table 1. ARIMA Models for Return Series of Diamond Bank

Model	Parameter				Akaike Information Criteria (AIC)
	φ_1	φ_2	θ_1	θ_2	
ARIMA(1, 1, 1)	0.3349^{***}		-0.0957		-11357.69
ARIMA(1, 1, 2)	-0.0476		0.2858	0.1093^{*}	-11360.79
ARIMA(2, 1, 1)	-0.5029^{***}	0.2199^{***}	0.7404^{***}		-11360.86

**** significance at 5% level ; * significance at 1% level*

Furthermore, Evidence from Ljung - Box Q-statistics shows that ARIMA (2, 1, 1) model is adequate at 5% level of significance given the Q-statistic at Lags 1, 4, 8 and 24, that is, Q(1) = 0.0084, Q(4) = 1.5075, Q(8) =6.3308 and Q(24) = 25.476 with corresponding (*P = .93*), *(P = .83)*, *(P = .61)* and *(P = .38),* respectively.

3.3 Identification heteroscedasticity in the return series of diamond bank

From Figs. 9 and 10, it could be observed that heteroscedasticity exists in the residual series of ARIMA (2, 1, 1) model since the lags 1, 2, 3, 15, 16, 20 of the ACF and Lags 1, 2, 3 and 15 of PACF are outside the significance bounds.

Also, Heteroscedasticity is said to exist in the residual series at lags 4, 8, 12, 16, 20 and 24 since the Portmanteau-Q statistics; Q(4) = 66.1, Q(8) = 85.9, Q(12) = 95.7, Q(16) = 133.9, Q(20) = 143.1 and Q(24) = 148.6 whose corresponding (*P = 1.53e-13*), (*P = 3.11e-15*), (*P = 3.89e-15*),(*P = .00*), (*P = .00*) and (*P = .00*) are all less than 5% level of significance.

Further evidence from Lagrange-Multiplier (LM) test statistics confirms that heteroscedasticity is present in residual series of ARIMA (2,1,1) model at lags 4, 8, 12, 16, 20 and 24 since the Lagrange Multiplier test statistics; LM(4) = 2021, LM(8) = 992, LM(12) = 651, LM(16) = 472, LM(20) = 373 and LM(24) = 307 whose corresponding (*P* = *.00*), (*P* = *.00*), (*P* = *.00*), (*P* = *.00*), (*P* = *.00*) and (*P* = *.00*) are all less than 5% level of significance.

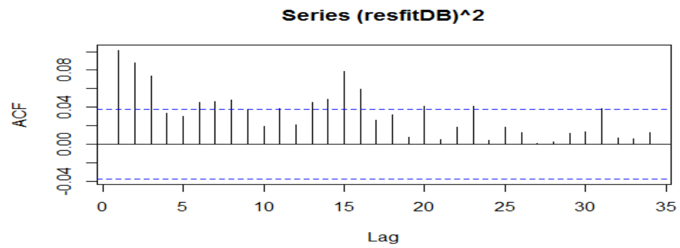

Fig. 9. ACF of squared residuals of ARIMA (2, 1, 1) model

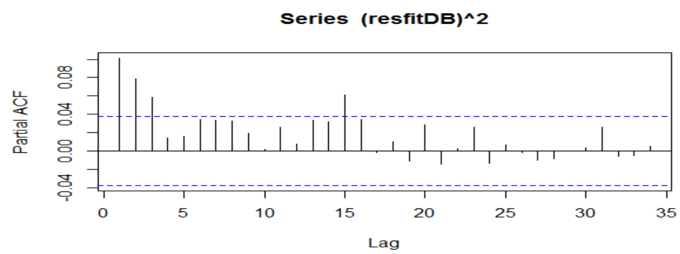

Fig. 10. PACF of squared residuals of ARIMA (2, 1, 1) model

3.4 Identification of outliers in the residual series of ARIMA (2, 1, 1) model fitted to the return series of diamond bank

Using the critical value, C = 4 and based on the condition n≥ 450, about seventeen (17) different outliers are identified to have contaminated the residual series of ARIMA (2, 1, 1) model; four (4) innovation outliers (IO), ten (10) additive outliers and three (3) temporary change. The outliers at a given time are indicated as follows: IO (t = 99), IO (t = 642), IO (t = 1671), IO (t = 1791), AO (t = 1656), AO (t = 1723), AO (t = 1739), AO (t = 1770), AO (t = 1843), AO (t = 2263), AO (t = 2281), AO (t = 2562), AO (t = 2626), TC (t = 98), AO (t = 2559), TC (t = 1667) and TC (t = 2554).

3.5 Building ARIMA (2, 1, 1) model for outlier adjusted return series of diamond bank

Having identified and ascertained that the return series of Diamond bank is outliers contaminated, the outliers effects are removed from the series to produce a new series that is outliers free and we refer to such series as outlier adjusted series. Also, ARIMA (2, 1, 1) model is fitted to the outlier adjusted series with the parameters all significant at 5% level [Table 2] and is found to be adequate at 5% level of significance given the Q-statistics at Lags 1, 4, 8 and 24, that is, Q(1) = 0.0498, Q(4) = 2.7683, Q(8) =9.3022 and Q(24) =32.272 with corresponding (*P* = *.82*), (*P* = *.60*), (*P* = *.32*) and (*P* = *.12*).

Table 2. ARIMA (2, 1, 1) model for outlier adjusted return series of diamond bank

Model	**Parameter**			**Akaike Information Criteria (AIC)**
	φ_1	φ_2	θ_1	
ARIMA (2, 1, 1)	-0.5215^{***}	0.2375^{***}	0.7750^{***}	-11622.05

**** significance at 5% level*

3.6 Identification of heteroscedasticity in the outlier adjusted return series of diamond bank

Considering the ACF and PACF of the squared residual series of ARIMA (2, 1, 1) model fitted to the outlier adjusted return series of Diamond bank, from Figs. 11 and 12, it could be observed that heteroscedasticity exists in the residual series of ARIMA (2, 1, 1) model since some lags of ACF and PACF are outside the significance bounds

Also, heteroscedasticity is said to exist in the residual series at lags 4, 8, 12, 16, 20 and 24 since the Portmanteau-Q statistics; Q(4) = 205, Q(8) = 298, Q(12) = 353, Q(16) = 443, Q(20) = 496 and Q(24) = 518 whose corresponding (*P = .00*), (*P = .00*), (*P = .00*), (*P = .00*), (*P = .00*) and (*P = .00*) are all less than 5% level of significance.

Further evidence from Lagrange-Multiplier (LM) test statistics confirms the presence of heteroscedasticity at lags 4, 8, 12, 16, 20 and 24 since the Lagrange Multiplier test statistics; LM(4) = 608.2, LM(8) = 291.4, LM(12) = 187.9, LM(16) = 137.3, LM(20) = 107.4 and LM(24) = 88.8 with corresponding (*P = .00*), (*P = .00*), (*P = .00*), (*P = .00*), (*P = 2.38e-14*) and (*P =1.14e-09*) are all less than 5% level of significance.

Fig. 11. ACF of squared residuals of ARIMA (2, 1, 1) model fitted to outlier adjusted return series of diamond bank

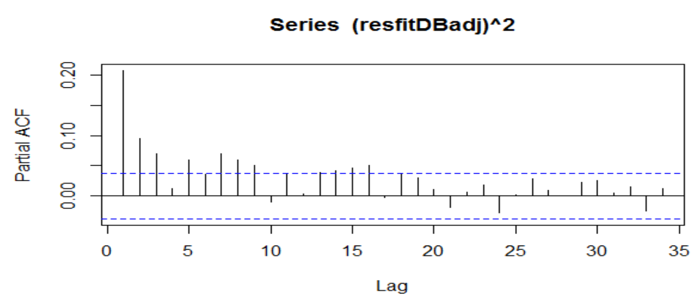

Fig. 12. PACF of squared residuals of ARIMA (2, 1, 1) model fitted to outlier adjusted return series of diamond bank

3.7 Effects of outliers on heteroscedasticity identification tools in the return series of diamond bank

Correlogram: Comparing the ACF and PACF of the squared residuals of ARIMA (2, 1, 1) model fitted the outlier contaminated return series of Diamond bank [Figs. 9 and 10] to the ACF and PACF of the squared residuals of ARIMA (2, 1, 1) model fitted the outlier adjusted return series of Diamond bank [Figs. 11 and 12], it is obvious that the significant lags in both ACF and PACF of squared residuals of the ARIMA (2, 1, 1) model fitted the outlier adjusted return series are increasing and more in number than those of the squared residuals of ARIMA (2, 1, 1) model fitted the outlier contaminated return series. Hence, it could be deduced that the presence of outliers hides heterosceadsticity detection in ACF and PACF of return series of Diamond bank.

Ljung-Box (Portmanteau) Q test: To Investigate the effects of outliers on the Ljung-Box (Portmanteau) Q-test, we compare the values of the Q- Statistic on the residuals of ARIMA (2, 1, 1) model fitted to return series contaminated with outliers to the values of the Q- Statistic on the residuals of ARIMA (2, 1, 1) model fitted to the outlier adjusted return series. From Table 3, using the outlier contaminated series as a reference point, we identified that the presence of outliers reduces the power of Ljung-Box test by 210.14%, 246.97%, 268.86%, 230.84%, 244.01% and 248.59% at lags 4, 8, 12, 16, 20 and 24, respectively. The implication is that, in the presence of outliers, the Ljung-Box test is distorted with its power becoming reduced and lower. Thus, the identification of true heteroscedasticity is hindered.

Table 3. Effects of outliers on Ljung-box (Portmanteau) Q test

Lag (order)	Value of Q-statistic on residual series of ARIMA (2, 1, 1) model fitted to returns series of diamond bank	Value of Q-statistic on residuals of ARIMA (2, 1, 1) model fitted to outlier adjusted return series of diamond bank	Average effect of outlier identified (%)
4	66.1	205	-210.14
8	85.9	298	-246.92
12	95.7	353	-268.86
16	133.9	443	-230.84
20	143.6	496	-244.01
24	148.6	518	-248.59

Lagrange Multiplier Test: To investigate the effects of outliers on the Lagrange Multiplier (LM) test, we compare the values of the LM test Statistic on the residuals of ARIMA (2, 1, 1) model fitted to return series of contaminated with outliers to the values of the LM test statistic on the residuals of ARIMA (2, 1, 1) model fitted to the outlier adjusted return series. From Table 4, using the outlier contaminated series as a reference point, we identified that the presence of outliers increases the power of Lagrange Multiplier test by 232.29%, 240.43%, 246.46%, 242.28%, 247.30% and 245.72% at lags 4, 8, 12, 16, 20 and 24, respectively. The implication is that, in the presence of outliers, the Lagrange Multiplier test is distorted with its power becoming increased and higher. Thus, spurious heteroscedasticity is detected when using Lagrange Multiplier test in the presence of outliers.

Table 4. Effects of outliers on lagrange multiplier LM test

Lag (order)	Value of LM on residual series of ARIMA (2, 1, 1) model fitted to returns series of diamond bank	Value of LM on residuals of ARIMA (2, 1, 1) model fitted to outlier adjusted return series of diamond bank	Average effect of outlier identified (%)
4	2021	608.2	232.29
8	992	291.4	240.43
12	651	187.9	246.46
16	472	137.9	242.28
20	373	107.4	247.30
24	307	88.8	245.72

3.8 Fidelity Bank

From Figs. 13 and 14, both ACF and PACF indicate that mixed model could be entertained. The following models, ARIMA (1, 1, 0), ARIMA (0, 1, 1), ARIMA (1, 1, 1), ARIMA (1, 1, 2) and ARIMA (2, 1, 1) are entertained tentatively.

From Table 5, ARIMA (1, 1, 1) model has the smallest AIC but one of its parameters is not significant. While ARIMA (1, 1, 2) model has the second smallest AIC but its parameters are not significant. However, ARIMA (1, 1, 0) model is selected based on the ground that its parameter is significant and nearest minimum AIC.

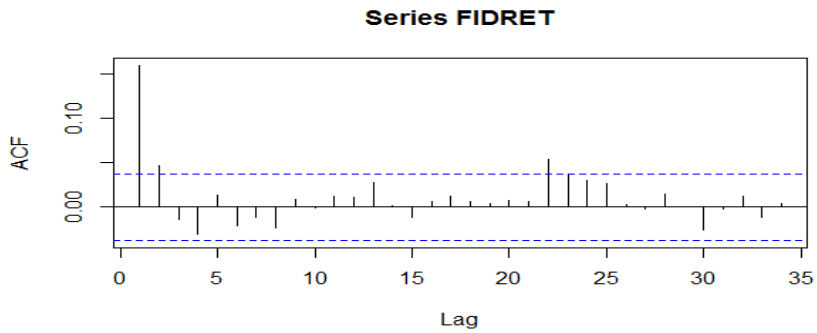

Fig. 13. ACF of return series of fidelity bank

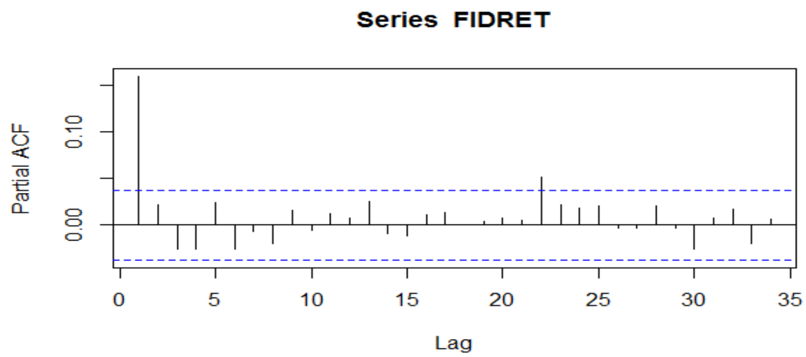

Fig. 14. PACF of return series of fidelity bank

Table 5. ARIMA models for return series of fidelity bank

Model	**Parameter**				**Akaike information criteria (AIC)**
	φ_1	φ_2	θ_1	θ_2	
ARIMA (1, 1, 0)	0.1606***				−11562.17
ARIMA (0, 1, 1)			0.1494***		−11559.28
ARIMA (1, 1, 1)	0.2569***		− 0.0986		−11563.16
ARIMA (1, 1, 2)	−0.0498		0.2071	0.0628	−11562.88
ARIMA (2, 1, 1)	−0.0721	0.0619	0.2288		−11561.98

**** significance at 5% level*

Furthermore, evidence from Ljung-Box Q-statistics shows that ARIMA(1,1,0) model is adequate at 5% level of significance given the Q-statistics at Lags 1, 4, 8 and 24, that is, Q(1) = 0.0376, Q(4) = 5.4261, Q(8) = 9.8001 and Q(24) = 23.379 with corresponding ($P = .85$), ($P = .25$), ($P = .28$) and ($P = .50$), respectively.

3.9 Identification of heteroscedasticity in the return series of fidelity bank

From Figs. 15 and 16, it could be observed that heteroscedasticity exists in the residual series of ARIMA (1, 1, 0) model since some lags of ACF and PACF are outside the significance bounds.

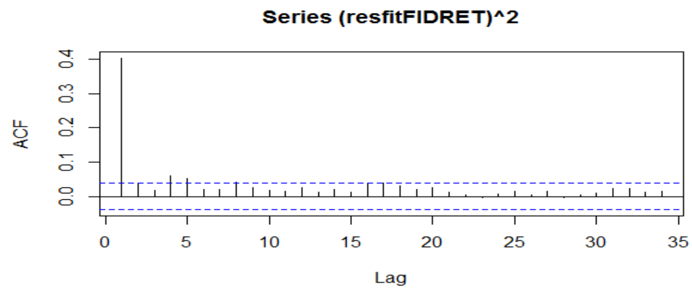

Fig. 15. ACF of the squared residuals of ARIMA (1, 1, 0) model

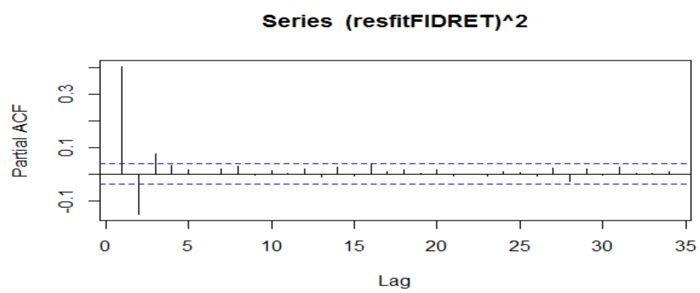

Fig. 16. PACF of the squared residuals of ARIMA (1, 1, 0) model

Heteroscedasticity is said to exist in the residual series at lags 4, 8, 12, 16, 20 and 24 since the Portmanteau-Q statistics, Q(4) = 450, Q(8) = 463, Q(12) = 468, Q(16) = 474, Q(20) = 483 and Q(24) = 484 with corresponding (P = .00), (P = .00), (P = .00), (P = .00), (P = .00) and (P = .00) are all less than 5% level of significance.

Also, evidence from Lagrange Multiplier (LM) test statistics confirms that heteroscedasticity is present at lags 4, 8, 12, 16, 20 and 24 since the Lagrange Multiplier test statistics, LM(4) = 1423, LM(8) = 704, LM(12) = 466, LM(16) = 347, LM(20) = 275 and LM(24) = 228 with corresponding (P = .00), (P = .00), (P = .00), (P = .00), (P = .00) and (P = .00) are all less than 5% level of significance.

3.10 Identification of outliers in the residual series of ARIMA (1, 1, 0) model fitted to the return series of fidelity bank

Considering the critical value, C = 4 and based on the condition that n ≥ 450, about sixteen (16) different outliers are identified to have contaminated the residuals series of ARIMA (1, 1, 0) model, two (2) innovation outliers (IO), five (5) additive outliers and nine (9) temporary change. The outliers at a given time are indicated as follows: IO (t = 1555), IO (t = 2292), AO (t = 1789), AO (t = 1841), AO (t = 2042), AO (t = 2539), AO (t = 2043), TC (t = 827), TC (t = 847), TC (t = 859), TC (t = 1665), TC (t = 1724), TC (t = 2263), TC (t = 2280), TC (t = 691) and TC (t = 950). However, in financial time series, it is assumed that the error is uncorrelated with its past value, and then all the outliers are classified as innovation outliers with a unified effect.

3.11 Building ARIMA (1, 1, 0) model for outlier adjusted return series of fidelity bank

ARIMA(1,1,0) model is fitted to the outlier adjusted series with its parameter significant at 5% level [Table 6] and is found to be adequate at 5% level of significance given the Q-statistics at Lags 1, 4, 8 and 24, that is, Q(1) = 0.0003, Q(4) = 4.2007, Q(8) =13.92 and Q(24) = 29.649 with corresponding (*P* = *.99*), (*P* = *.38*), (*P* = *.09*) and (*P* = *.20*).

Table 6. ARIMA (1, 1, 0) model for outlier adjusted return series of fidelity bank

Model	Parameter (φ)	Akaike information criteria
ARIMA(1,1,0)	0.1715***	−11954.67

**** significance at 5% level*

3.12 Identification of heteroscedasticity in Outlier adjusted return series of fidelity bank

In Figs. 17 and 18, it could be observed that heteroscedasticity exists in the residual series of ARIMA (1, 1, 0) model since several the lags of ACF and PACF are outside the significance bounds;

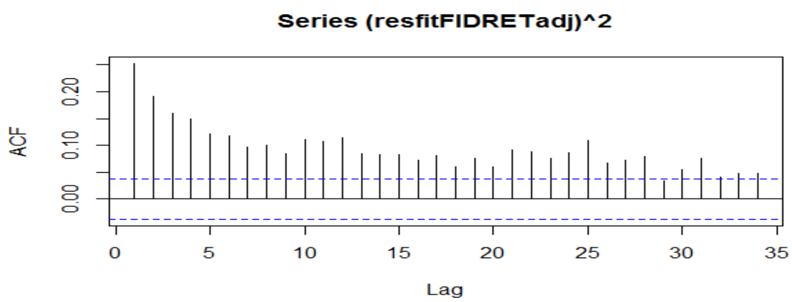

Fig. 17. ACF of squared residuals of ARIMA (1, 1, 0) model fitted to outlier adjusted return series of fidelity bank

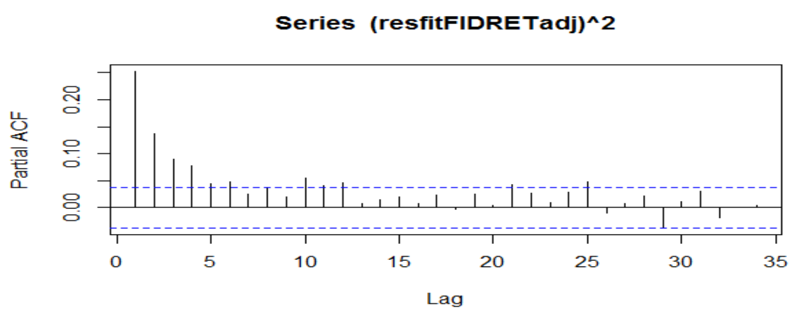

Fig. 18. PACF of squared residuals of ARIMA (1, 1, 0) model fitted to outlier adjusted return series of fidelity bank

Also, heteroscedasticity is said to exist in the residual series at lags 4, 8, 12, 16, 20 and 24 since the Portmanteau-Q statistics, Q(4) = 399, Q(8) = 527, Q(12) = 646, Q(16) = 715, Q(20) = 768 and Q(24) = 847 with corresponding (*P* = *.00*), (*P* = *.00*), (*P* = *.00*), (*P* = *.00*) and (*P* = *.00*) are all less than 5% level of significance.

More evidence from Lagrange-Multiplier (LM) test statistics confirms that heteroscedasticity is present in residual series of ARIMA (1, 1, 0) model fitted to outlier adjusted return series at lags 4, 8, 12, 16, 20 and 24 since the Lagrange Multiplier test statistics, LM(4) = 372.7, LM(8) = 177.3, LM(12) = 114.5, LM(16) = 84.4, LM(20) = 66.7 and LM(24) = 54.7 with corresponding (*P* = *.00*), (*P* = *.00*), (*P* = *.00*), (*P* =*1.10e-11*), (*P* =*3.18e-07*) and (*P* = *2.17e-04*) are all less than 5% level of significance.

3.13 Effects of outliers on heteroscedasticity identification tools in the return series of fidelity bank

Correlogram: Comparing the ACF and PACF of the squared residuals of ARIMA (1, 1, 0) model fitted the outlier contaminated return series of [Figs. 15 and 16] to the ACF and PACF of the squared residuals of ARIMA (1, 1, 0) model fitted the outlier adjusted return series of [Figs. 17 and 18], it is obvious that the significant lags in both ACF and PACF of squared residuals of the ARIMA(1, 1, 0) model fitted the outlier adjusted return series of are increasing and more in number than those of the squared residuals of ARIMA (1, 1, 0) model fitted the outlier contaminated return series. Hence, it could be deduced that the presence of outliers hides heterosceadsticity detection in ACF and PACF of return series of Fidelity bank.

Ljung-Box (Portmanteau) Q test: From Table 7, using the outlier contaminated series as a reference point, we identified that the presence of outliers reduces the power of Ljung-Box test by 13.82%, 38.03%, 50.84% , 59.01and 74.79% at lags 8, 12, 16, 20 and 24, respectively with exception at lag 4 where the power of Ljung-Box test is increased by 24.67%. The implication is that, in the presence of outliers, the Ljung- Box test is distorted with its power becoming reduced and lower. Thus, the identification of true heteroscedasticity is hindered.

Table 7. Effects of Outliers on Ljung-Box (Portmanteau) Q test

Lag (order)	Value of Q-statistic on residual series of ARIMA (1, 1, 0) model fitted to returns series of fidelity bank	Value of Q-statistic on residuals of ARIMA (1, 1, 0) model fitted to outlier adjusted return series of fidelity bank	Average effect of outlier identified (%)
4	450	399	24.67
8	463	527	-13.82
12	468	646	-38.03
16	474	715	-50.84
20	483	768	-59.01
24	484	847	-74.79

Lagrange Multiplier Test: From Table 8, using the outlier contaminated series as a reference point, we identified that the presence of outliers increases the power of Lagrange Multiplier test by 73.81%, 74.82%, 75.43%, 75.65%, 75.75% and 76.01% at lags 4, 8, 12, 16, 20 and 24, respectively. The implication is that, in the presence of outliers, the Lagrange Multiplier test is distorted with its power becoming increased and higher. Thus, spurious heteroscedasticity is detected when using Lagrange Multiplier test in the presence of outliers.

Table 8. Effects of outliers on lagrange multiplier LM test

Lag (order)	Value of LM on residual series of ARIMA (1, 1, 0) model fitted to returns series of fidelity bank	Value of LM on residuals of ARIMA (1, 1, 0) model fitted to outlier adjusted return series of fidelity bank	Average effect of outlier identified (%)
4	1423	372.7	73.81
8	704	177.3	74.82
12	466	114.5	75.43
16	347	84.4	75.65
20	275	66.7	75.75
24	228	54.7	76.01

3.14 Skye bank

From Figs. 19 and 20, both ACF and PACF indicate that mixed model could be entertained. The following models, ARIMA (1, 1, 0), ARIMA (0, 1, 1) and ARIMA (1, 1, 1) are entertained tentatively.

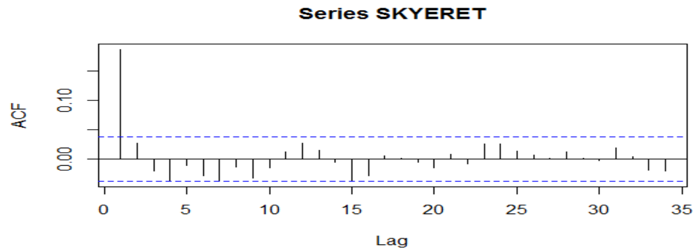

Fig. 19. ACF of return series of Skye bank

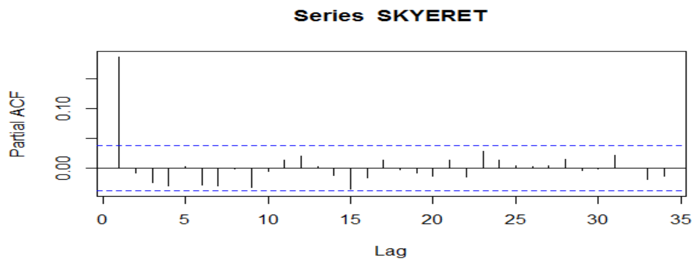

Fig. 20. ACF of return series of Skye bank

From Table 9, the ARIMA (1, 1, 0) model is selected based on the ground of smallest AIC and the significance of the parameters.

Table 9. ARIMA models for return series of Skye bank

Model	Parameter		Akaike information criteria (AIC)
	φ_1	θ_1	
ARIMA (1, 1, 0)	0.1874***		−10713.39
ARIMA (0, 1, 1)		0.1827***	−10711.03
ARIMA (1, 1, 1)	0.1522	0.0364	−10711.54

**** significance at 5% level*

Moreover, evidence from Ljung - Box Q-statistics shows that ARIMA (1, 1, 0) model is adequate at 5% level of significance given the Q-statistics at Lags 1, 4, 8 and 24, that is, Q(1) = 0.0050, Q(4) = 4.1838, Q(8) = 8.2689 and Q(24) = 22.469 with corresponding ($P = .94$), ($P = .38$), ($P =.41$) and (P = .55), respectively.

3.15 Identification of heteroscedasticity in the return series of Skye bank

From the ACF and PACF of the squared residual series of ARIMA (1, 1, 0) model in Figs. 21 and 22, it could be observed that heteroscedasticity exists in the residual series of ARIMA (1, 1, 0) model since the first lags of ACF and PACF are outside the significance bounds.

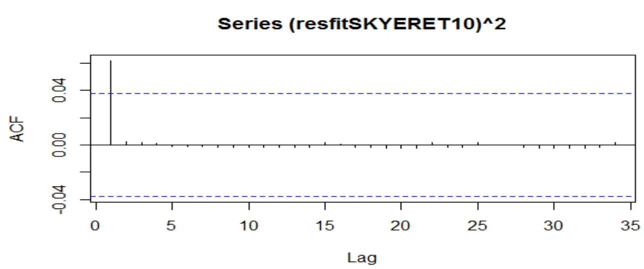

Fig. 21. ACF of the squared residuals of ARIMA (1, 1, 0) model

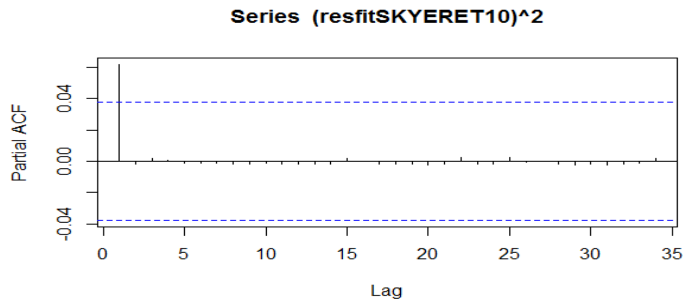

Fig. 22. PACF of the squared residuals of ARIMA (1, 1, 0) model

For the residuals of ARIMA (1, 1, 0) model at lags 4, 8, 12, 16, 20 and 24, and the Portmanteau-Q statistics; Q(4) = 10.3, Q(8) = 10.3, Q(12) = 10.3, Q(16) = 10.4, Q(20) = 10.4 and Q(24) = 10.5 with corresponding (*P = .04*), (*P = .24*), (*P = .59*), (*P = .85*), (*P = .96*) and (*P = .99*). It is observed that heteroscedasticity exists only at lag 4 at 5% level of significance.

Also, evidence from Lagrange-Multiplier (LM) test statistics confirms that heteroscedasticity is present in residual series of ARIMA (1, 1, 0) model at lags 4, 8, 12, 16, 20 and 24 since the Lagrange Multiplier test statistics, LM(4) = 57956, LM(8) = 28852, LM(12) = 19141, LM(16) = 14284, LM(20) = 11371 and LM(24) = 9423 with corresponding (*P = .00*), (*P = .00*), (*P = .00*), (*P = .00*), (*P = .00*) and (*P = .00*) are all less than 5% level of significance.

3.16 Identification of outliers in the residual Series of ARIMA (1, 1, 0) model fitted to the return series of Skye bank

Using the critical value, C = 4 and based on the condition $n \geq 450$, about twenty six (26) different outliers were identified to have contaminated the residuals series of ARIMA(1,1,0) model, six (6) innovation outliers (IO), six (6) additive outliers and fourteen (14) temporary change (TC). The outliers at a given time are indicated as follows: IO (t = 211), IO (t = 1841), IO(t = 1843), IO(t = 2178), IO(t = 2263), IO(t = 2314), AO (t = 210), AO (t = 1726), AO (t = 1984), AO (t = 2281), AO (t = 2414), AO(t = 2456), TC (t = 209), TC (t = 740), TC (t = 742), TC (t = 827), TC (t = 1723), TC (t = 2311), TC (t = 2381), TC (t = 2468), TC (t = 2590), TC (t = 2592), TC (t = 2599), TC (t = 212), TC (t = 741) and TC (t = 2589). However, in financial time series, it is assumed that the error is uncorrelated with its past value as such all the outliers are classified as innovation outliers with a unified effect.

3.17 Building ARIMA (1, 1, 0) model for outlier adjusted return series of skye bank

ARIMA (1, 1, 0) model is fitted to the outlier adjusted series with the parameter significant [Table 10] and is found to be adequate at 5% level given the Q-statistics at Lags 1, 4, 8 and 24, that is, Q(1) = 0.1224, Q(4) = 3.7952, Q(8) =7.7095 and Q(24) =22.691 with corresponding (*P =0.73*), (*P = 0.43*), (*P = 0.46*) and (*P = 0.54*), respectively.

Table 10. ARIMA (1, 1, 0) model for outlier adjusted return series of Skye bank

Model	Parameter (φ)	Akaike information criteria
ARIMA(1,1,0)	0.2425^{***}	-11692.3

*** *significance at 5%*

3.18 Identification of heteroscedasticity in outlier adjusted return series of skye bank

In Figs. 23 and 24, it could be observed that heteroscedasticity exists in the residual series of ARIMA (1, 1, 0) model since all the lags of the ACF and some lags of PACF are outside the significance bounds.

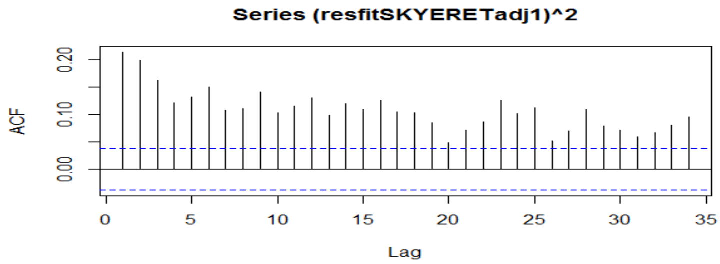

Fig. 23. ACF of squared residuals of ARIMA (1, 1, 0) model fitted to outlier adjusted return series of Skye bank

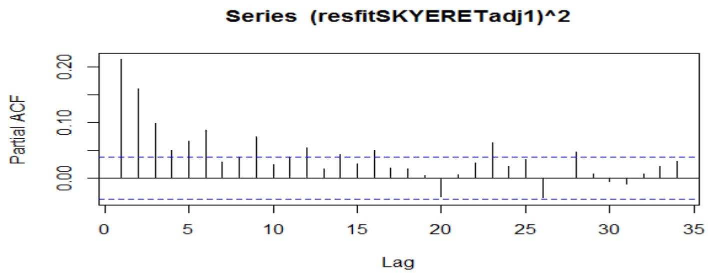

Fig. 24. PACF of Squared Residuals of ARIMA (1, 1, 0) model fitted to outlier adjusted return series of Skye bank

Heteroscedasticity is said to exist in the residual series at lags 4, 8, 12, 16, 20 and 24 since the Portmanteau-Q statistics, Q(4) = 341, Q(8) = 514, Q(12) = 678, Q(16) = 817, Q(20) = 902 and Q(24) = 1006 with corresponding ($P = .00$), ($P = .00$), ($P = .00$), ($P = .00$), ($P = .00$) and ($P = .00$) are all less than 5% level of significance.

Also, evidence from Lagrange-Multiplier (LM) test statistics confirms that heteroscedasticity is present in residual series of ARIMA (1, 1, 0) model fitted to outlier adjusted return series at lags 4, 8, 12, 16, 20 and 24 since the Lagrange Multiplier test statistics, LM(4) = 468.4, LM(8) = 219.7, LM(12) = 140.8, LM(16) = 102.7, LM(20) = 80.8 and LM(24) = 65.4 with corresponding ($P = .00$), ($P = .00$), ($P = .00$), ($P = 4.0e\text{-}15$), ($P = 1.38e\text{-}09$) and ($P = 6.09e\text{-}06$) are all less than 5% level of significance.

3.19 Effects of outliers on heteroscedasticity identification tools in the return series of Skye bank

Correlogram: Comparing the ACF and PACF of the squared residuals of ARIMA(1,1,0) model fitted the outlier contaminated return series of Skye bank [Figs. 21 and 22] to the ACF and PACF of the squared residuals of ARIMA(1, 1, 0) model fitted the outlier adjusted return series [Figs. 23 and 24], it is obvious that the significant lags in both ACF and PACF of squared residuals of the ARIMA (1, 1, 0) model fitted the outlier adjusted return series are increasing and more in number than those of the squared residuals of ARIMA (1, 1, 0) model fitted the outlier contaminated return series. Hence, it could be deduced that the presence of outliers hides heterosceadsticity detection in ACF and PACF of return series of Skye bank.

Ljung-Box (Portmanteau) Q test: From Table 11, using the outlier contaminated series as a reference point, we identified that the presence of outliers reduces the power of Ljung-Box test by 3210.68%, 4890.29%, 6482.52%, 7755.77%, 8573.08 and 98.96% at lags 4, 8, 12, 16, 20 and 24, respectively. The implication is that, in the presence of outliers, the Ljung- Box test is distorted with its power becoming reduced and lower. Thus, the identification of true heteroscedasticity is hindered.

Table 11. Effects of outliers on Ljung-box (Portmanteau) Q test

Lag (order)	Value of Q-statistic on residual series of ARIMA (1 ,1 ,0) model fitted to returns series of Skye bank	Value of Q-statistic on residuals of ARIMA(1 ,1, 0) model fitted to outlier adjusted return series of Skye bank	Average effect of outlier identified (%)
4	10.3	341	-3210.68
8	10.3	514	-4890.29
12	10.3	678	-6482.52
16	10.4	817	-7755.77
20	10.4	902	-8573.08
24	10.5	1006	-9480. 95

3.20 Lagrange multiplier test

From Table 12, using the outlier contaminated series as a reference point, we identified that the presence of outliers increases the power of Lagrange Multiplier test by 99.19%,99.24%, 99.26%, 99.28%, 99.29 and 99.31% at lags 4, 8, 12, 16, 20 and 24, respectively. The implication is that, in the presence of outliers, the Lagrange Multiplier test is distorted with its power becoming increased and higher. Thus, spurious heteroscedasticity is detected when using Lagrange Multiplier test in the presence of outliers.

Table 12. Effects of outliers on Lagrange multiplier LM test

Lag (order)	Value of LM on residual series of ARIMA(1,1,0) model fitted to returns series of Skye bank	Value of LM on residuals of ARIMA(1,1,0) model fitted to outlier adjusted return series of Skye bank	Average effect of outlier identified (%)
4	57956	468.4	99.19
8	28852	219.7	99.24
12	19141	140.8	99.26
16	14284	102.7	99.28
20	11371	80.8	99.29
24	9423	65.4	99.31

4 Conclusion

So far, ARIMA (2, 1, 1), ARIMA (1, 1, 0) and ARIMA (1, 1, 0) models were identified and successfully fitted to the share price returns series of Diamond Bank, Fidelity Bank and Skye bank, respectively. The series of the three banks were found to be contaminated with several outliers. Having removed the effects of outliers from the series and for the purpose of argument, ARIMA (2, 1, 1), ARIMA (1, 1, 0) and ARIMA (1, 1, 0) models were fitted to the outlier-adjusted series of the three respective banks. Particularly, heteroscedasticity was detected in both outlier contaminated and outlier adjusted series of the respective banks using correlogram, Ljung-Box test and Lagrange Multiplier. Our findings revealed that outliers distort, hamper and hide or exaggerate the detection of true heteroscedasticity in returns series of the banks under study. Hence, it could be deduced that, in order to detect and identify the true heteroscedasticity in discrete-time series, it is important to take into consideration the presence of outliers. Furthermore, this study could be extended to cover the effects of outliers on parameters estimation in heteroscedastic models.

Competing Interests

Authors have declared that no competing interests exist.

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