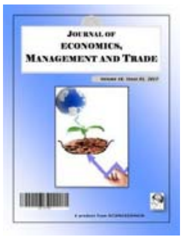

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Optimal Allocation of Funds for Loans Using Karmarkar's Algorithm: Capital Rural Bank, Sunyani-Ghana

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Authors' contributions

This work was carried out in collaboration between all authors. Author AD designed the study, performed the statistical analysis, wrote the protocol and first draft of the manuscript. Authors KD and JA managed the analyses of the study. Authors PG and NW managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

In Ghana, the banking industry is now characterised by increasing competition and innovation. This has made most of the banks to adopt a scientific approach to improve the quality of their loan structure. The decline of relevant portfolio planning models especially in Ghana is attributed mainly to the evolving dynamics of the Ghanaian banking industry where the regulatory controls have changed with a high frequency. Due to the model used in allocating funds to various loan types, a lot of banks had suffered substantial losses from some bad loans in their portfolio. As a result, most banks are not able to maximize their profit on loans due to poor allocation of funds. The purpose of

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this study is to develop a linear programming model using the Karmarkar's projective scaling algorithm to help Capital Rural Bank Limited to maximise their profit on loans. The results from the model showed that Capital Rural Bank Limited would be making annual loan turn-over of **GH¢5,961,300.00** which is 61.3% more than the established previously. From the study, it was further realised that policy directions mostly influence the optimal solution and not probability of bad debt.

Keywords: Linear programming; Karmarkar's algorithm; maximize profit.

1. INTRODUCTION

Lending to firms is an essential business activity for every commercial bank. This means that loans portfolio management is the necessary activity for getting the maximum return and minimum risk from bank loans. It is fundamental to the safety and sustainability of banks. Most loan managers concentrate their effort on how to effectively approve loans and carefully monitor loan performance. It is therefore vital and prudent for financial establishments like banks to maximise the return on their loan portfolio [1,2]. According to Cohen and Hammer, what makes the task of funds allocation difficult is finding an appropriate balance between three desirable objectives in loans portfolio management [3]. These objectives are; profitability, liquidity and safety. Banks should decide on the distribution of its capital among the various types of loans, which differ in duration and risk and are affected by the environment, the borrowers' deposits at the given bank as well as other factors (Klaassen) [4]. Besides, banking loan decisions require the use of large data and substantial processing time to be able to serve a large number of variables and a variety of different cases related to different customers. In this paper, we study how a rural bank in Ghana can optimise the allocation of funds for loans.

Linear programming is frequently applied in portfolio management. To maximise profit on loans, funds must be optimally allocated to the different loan types and linear programming models are the suitable models for optimal allocation of resources. Dantzig [5] introduced the simplex algorithm in 1947 was among the pioneers to use it to optimally solve linear programming problem and thus can be used to optimally allocate funds to different loan types [5]. Karmarkar introduced an interior point algorithm for solving linear programming problems. It was the first reasonably efficient algorithm that solves these problems in polynomial time. It does not follow the boundary of the feasible set as in the simplex method, but moves through the interior of the feasible region, improving the approximation of the optimal solution [6,7,8].

According to Prosper (2011), Panos and Mauricio (1996) explained how interior point methods, originally invented in the context of linear programming, have found a much broader range of applications, including global optimization problems that arise in engineering, computer science, operations research, and other disciplines. Quintana et al. [9], stated that since Karmarkar's first successful interior-point algorithm for linear programming in 1984, the interest and consequently the numbers of publications in the area have increased tremendously. They reviewed and classified significant publications on interior point methods, on the practical implementation of the most successful interior-point algorithms and on their applications to power systems optimisation problems [9].

Ferris and Philpott [10], described how Karmarkar's polynomial-time algorithm for linear programming significantly outperform the simplex method. The described many numerical experiments carried out by other workers in the field which show a much smaller iteration count than the simplex method but larger computational times. Some had shown that, by using advanced numerical linear algebra and heuristics to exploit the problem structure, it is possible occasionally to beat the simplex method even in terms of computation time [10].

Kojima [11] reported that Karmarkar interior-point method has been successfully extended in the field of continuous optimization to convex quadratic programs, semi-definite programs, and more general convex problems, while, in the field of discrete optimization, it has been playing an important role in terms of the semi-definite programming relaxation of 0-1 integer and nonconvex quadratic programs [11].

Todd [12], showed a variant of Karmarkar's projective algorithm for linear programming can be viewed as following the approach of Dantzig-

Wolfe decomposition. At each iteration, the current primal feasible solution generates prices which are used to form a simple subproblem. The solution to the subproblem is then incorporated into the currently feasible solution. With a suitable choice of step size a constant reduction in potential function is achieved at each iteration [12].

Monteiro [13], analyzed the convergence and boundary behaviour of the continuous trajectories of the vector field induced by the projective scaling algorithm as applied to (possibly degenerate) linear programming problems in Karmarkar's standard form. They showed that a projective scaling trajectory tends to an optimal solution which in general depends on the starting point. When the optimal solution is unique, they prove that all projective scaling trajectories approach the optimal solution through the same asymptotic direction. The analysis was based on the affine scaling trajectories for the homogeneous standard form that arises from Karmarkar's standard form by removing the unique non-homogeneous constraint [13].

Bayer and Lagarias (1989), described a geometric structure underlying Karmarkar's projective scaling algorithm for solving linear programming problems. A basic feature of the projective scaling algorithm is a vector field depending on the objective function which is defined on the interior of the polytope of feasible solutions of the linear program. The geometric structure studied is the set of trajectories obtained by integrating this vector field, which we call P-trajectories. They also study a related vector field, the affine scaling vector field, and its associated trajectories, called A-trajectories. The affine scaling vector field is associated to another linear programming algorithm, the affine scaling algorithm. Affine and projective scaling vector fields are each defined for linear programs of a special form, called strict standard form and canonical form, respectively. This derives basic properties of P-trajectories and A-trajectories. It reviews the projective and affine scaling algorithms, defines the projective and affine scaling vector fields, and gives differential equations for P-trajectories and A-trajectories. It shows that projective transformations map P-trajectories into P-trajectories. It presents Karmarkar's interpretation of A-trajectories as steepest descent paths of the objective function with respect to the Riemannian geometry restricted to the relative interior of the polytope of feasible solutions. In this paper we convert a standard maximization problem of a bank's loan portfolio allocation to Karmarkar form and solve the resulting loan portfolio optimization to optimality and further provide detail analysis of the loan allocation structure [14,15].

2. METHODS

Karmarkar's projective scaling method, also known as *Karmarkar's interior point LP algorithm*, starts with a trial solution and shoots it towards the optimum solution.

This algorithm addresses LP problem of the form:

Minimize Z = C^T X

Subject to

$$AX = 0$$
$$\sum_{i=1}^{n} x_i = 1$$
$$X \geq 0$$

where A is a *m* x *n* matrix of rank *m*, C^T is *1* x *n* vector,

X $= (x_1,\ldots,\ x_n)^T,\ n \geq 2$ and A and C are all real.

2.1 Converting Linear Programming Problem to Karmarkar's Form

In converting the LP problem:

Maximize Z = C^T X

Subject to

$$AX \leq b$$
$$X \geq 0$$

to Karmarkar's Form, the following steps are used:

Step 1:

Write the dual of the given primal problem

Minimize Z = bW

Subject to

$$A^T W \geq C^T$$
$$W \geq 0$$

Step 2:

Introduce Slack and Surplus variables to primal and dual problems

Combine these two (2) problems

Step 3:

Introduce a bounding constraint $\sum x_i + \sum w_i \leq K$, where *K* should be sufficiently large to include all feasible solutions of original problem

Introduce a slack variable in the bounding constraint and obtain

$$\sum x_i + \sum w_i + s = K$$

Step 4:

Introduce a dummy variable d (subject to condition d = 1) to homogenize the constraints and replace the equations

$\sum x_i + \sum w_i + s = K$ and $d = 1$ with the following equations:

$$\sum x_i + \sum w_i + s - Kd = 0 \text{ and}$$
$$\sum x_i + \sum w_i + s + d = K + 1$$

Step 5:

Introduce the following transformations so as to obtain one on the RHS of the last equation:

$$x_i = (K+1)y_i, i = 1,2,....m+n$$

$$w_i = (K+1)y_{m+n+i}, i = 1,2,...m+n$$

$$s = (K+1)y_{2m+2n+1}$$

$$d = (K+1)y_{2m+2n+2}$$

Step 6:

Introduce an artificial variable $y_{2m+2n+3}$ (to be minimized) in all the equations such that the sum of the coefficients in each homogenous equation is **zero** and coefficients of the artificial variable in the last equation is **one**. (Dhamija, A.K.) [16]

The general Karmarkar's form is as shown below:

Minimize M = C^T V

Subject to

AV = 0

$$\sum_{i=1}^{n} V_i = 1$$

V ≥ 0

2.2 Steps of Karmarkar's Algorithm

Step 1:

Compute $r = \sqrt{\frac{1}{n(n-1)}}$ and $\alpha = \frac{n-1}{3n}$.

Put *k* = 0 and let $V_0 = (1/n,....,1/n)^T$

Step 2:

(a) Let $Y_0 = V_0$

$$D_0 = diag\,(V_0)$$

$$P = \begin{bmatrix} AD_0 \\ 1 \end{bmatrix}, \quad 1 = [1 \;\; , \;\; . \;\; . \;\; . \;\; , \;\; 1]$$

$$\overline{C} = C^T D_0$$

Compute $C_p = \left[I - P^T (PP^T)^{-1} P\right] \overline{C}^T$

If , $C_p = 0$, any feasible solution becomes an optimal solution. Stop

(b) Otherwise compute

$$Y_{new} = Y_0 - \alpha r \frac{C_p}{\|C_p\|}$$

$$V_1 = \frac{D_0 Y_{new}}{1 D_0 Y_{new}}$$

$$M = C^T V_1$$

Step 3:

Increase *k by* one and repeat *Step 1* until the objective function (*M*) value is less than or equal to zero.

3. DATA PRESENTATION AND ANALYSIS

Capital Rural Bank provides a flexible loan payment term for all the types of loans. The

Management Board of the bank, that was recording bad debts on the loans and wanted a restructuring of the loan types. The data supplied by the bank is summarized in Table 1.

Table 1. Loans types, interest rate and probability of bad debt

S/N	Type of loan	Interest rate	Probability of bad debt
1	Commercial	0.39	0.020
2	Funeral	0.36	0.030
3	Salary	0.36	0.10
4	Susu	0.36	0.055
5	Agriculture	0.36	0.150
6	Housing	0.30	0.075

Source: Capital Rural Bank Limited, Sunyani

Table 2 is the constructed from Table 1, columns 2, 5 and 6 indicate the variable for loan amounts, formulas for amount of bad debt and profit respectively.

Capital Rural Bank Limited decided on a loan policy with an amount of GH¢20,000,000.00.

The policy details for the type of loans are:

- Salary, Funeral and Commercial loans should be at most 60% of the total funds.
- To assist people in the area (Sunyani Metropolis) to undertake more housing projects, the Housing loan should be at most 50% of Salary and Funeral loans.
- The sum of Susu and Agriculture loans must not be more than 40% of Commercial and Housing loans.
- The sum of the Agriculture and Funeral loans should be at most 15% of the total funds.
- The overall ratio for bad debts in all loans should not exceed 4.5%.

3.1 Formulation of Linear Programming (LP) Problem

The objective function is given by interest on profit amount less bad debt amount and the formula is

$$Z = \sum_{i=1}^{6} I_i (1 - B_i) x_i - \sum_{i=1}^{6} B_i x_i$$

$$Z = 0.3622x_1 + 0.3192x_2 + 0.3464x_3 + 0.2852x_4 + 0.1560x_5 + 0.2025x_6$$

Subject to the following policy constraints:

- Total funds available for disbursement is GH¢20,000,000.00

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 20$$

- Salary, Funeral and Commercial loans should be at most 60% of the total funds

$$x_1 + x_2 + x_3 \le 12$$

- Housing loan should be at most 50% of Salary and Funeral loans.

$$-0.5x_2 - 0.5x_3 + x_6 \le 0$$

- The sum of Susu and Agriculture loans must not be more than 40% of Commercial and Housing loans.

$$-0.4x_1 + x_4 + x_5 - 0.4x_6 \le 0$$

- The sum of the Agriculture and Funeral loans should be at most 15% of the total funds.

$$x_2 + x_5 \le 3$$

- The overall ratio for bad debts in all loans should not exceed 0.045

$$-0.025x_1 - 0.015x_2 - 0.035x_3 + 0.010x_4 + 0.105x_5 + 0.030x_6 \le 0$$

- Non-negativity

$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, x_4 \ge 0, x_5 \ge 0, x_6 \ge 0$$

The loan policy model is the Standard Linear Programming maximization problem below:

Maximize

$$Z = 0.3622x_1 + 0.3192x_2 + 0.3464x_3 + 0.2852x_4 + 0.1560x_5 + 0.2025x_6$$

Subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 20$$
$$x_1 + x_2 + x_3 \le 12$$
$$-0.5x_2 - 0.5x_3 + x_6 \le 0$$
$$-0.4x_1 + x_4 + x_5 - 0.4x_6 \le 0$$
$$x_2 + x_5 \le 3$$
$$-0.025x_1 - 0.015x_2 - 0.035x_3 + 0.010x_4 + 0.105x_5 + 0.030x_6 \le 0$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

Table 2. Formulation of bad debt and profit amounts

Loan type	Loan amt (x)	Interest rate (I)	Probability of bad debt (B)	Bad debt amt (Bx)	Profit amt $(1-B)x$
Commercial	x_1	0.39	0.020	0.020 x_1	0.980 x_1
Funeral	x_2	0.36	0.030	0.030 x_2	0.970 x_2
Salary	x_3	0.36	0.010	0.010 x_3	0.990 x_3
Susu	x_4	0.36	0.055	0.055 x_4	0.945 x_4
Agriculture	x_5	0.36	0.150	0.150 x_5	0.850 x_5
Housing	x_6	0.30	0.075	0.075 x_6	0.925 x_6

Source: Anthony Donkor (obtained from Table 1)

3.2 Converting the Linear Programming Problem into Karmarkar's Form

The Standard Linear Programming form is as shown below:

Maximize

$$Z = 0.3622x_1 + 0.3192x_2 + 0.3464x_3 + 0.2852x_4 + 0.1560x_5 + 0.2025x_6$$

Subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 20$$
$$x_1 + x_2 + x_3 \le 12$$
$$-0.5x_2 - 0.5x_3 + x_6 \le 0$$
$$-0.4x_1 + x_4 + x_5 - 0.4x_6 \le 0$$
$$x_2 + x_5 \le 3$$

$$-0.025x_1 - 0.015x_2 - 0.035x_3 + 0.010x_4 + 0.105x_5 + 0.030x_6 \le 0$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

- Writing the dual of the given primal problem:

Minimize $20w_1 + 12w_2 + 3w_5$

Subject to :

$$w_1 + w_2 - 0.4w_4 - 0.025w_6 \ge 0.3622$$
$$w_1 + w_2 - 0.5w_3 + w_5 - 0.015w_6 \ge 0.3192$$
$$w_1 + w_2 - 0.5w_3 - 0.035w_6 \ge 0.3464$$
$$w_1 + w_4 + 0.010w_6 \ge 0.2852$$
$$w_1 + w_4 + w_5 + 0.105w_6 \ge 0.1560$$
$$w_1 + w_3 - 0.4w_4 + 0.030w_6 \ge 0.2025$$
$$w_1, w_2, w_3, w_4, w_5, w_6 \ge 0$$

By the procedure explained in section 3 and illustrated in appendix A, the primal and dual linear programming problems are converted to the Karmarkar format to obtain

Minimize $M = y_{27}$

Subject to:

$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 - 20y_{26} + 13y_{27} = 0$$
$$y_1 + y_2 + y_3 + y_8 - 12y_{26} + 8y_{27} = 0$$
$$-0.5y_2 - 0.5y_3 + y_6 + y_9 - y_{27} = 0$$
$$-0.4y_1 + y_4 + y_5 - 0.4y_6 + y_{10} - 2.2y_{27} = 0$$
$$y_2 + y_5 + y_{11} - 3y_{26} = 0$$
$$-0.025y_1 - 0.015y_2 - 0.035y_3 + 0.010y_4 + 0.105y_5 + 0.030y_6 + y_{12} - 1.07y_{27} = 0$$
$$y_{13} + y_{14} - 0.4y_{16} - 0.025y_{18} - y_{19} - 0.3622y_{26} - 0.2128y_{27} = 0$$

$$y_{13} + y_{14} - 0.5y_{15} + y_{17} - 0.015y_{18} - y_{20} - 0.3192y_{26} - 1.1658y_{27} = 0$$

$$y_{13} + y_{14} - 0.5y_{15} - 0.035y_{18} - y_{21} - 0.3464y_{26} - 0.1186y_{27} = 0$$

$$y_{13} + y_{16} + 0.010y_{18} - y_{22} - 0.2852y_{26} - 0.7248y_{27} = 0$$

$$y_{13} + y_{16} + y_{17} + 0.105y_{18} - y_{23} - 0.1560y_{26} - 1.949y_{27} = 0$$

$$y_{13} + y_{15} - 0.4y_{16} + 0.030y_{18} - y_{24} - 0.2025y_{26} + 1.5725y_{27} = 0$$

$$0.3622y_1 + 0.3192y_2 + 0.3464y_3 + 0.2852y_4 + 0.1560y_5 + 0.2025y_6 - 20y_{13} - 12y_{14} - 3y_{17} + 33.3285y_{27} = 0$$

$$\sum_{i=1}^{25} y_i - 100y_{26} + 75y_{27} = 0$$

$$\sum_{i=1}^{27} y_i = 1$$

$y_i \geq 0$, $i = 1,2,3,...,\ 27$

3.3 Computational Procedure and Results

Using the Karmarker methodology in section 3, programming code was written in MATLAB using a tolerance of 1.0×10^{-15} .

(IPentium M, 1.80GHz, 0.99GB RAM). The program converged at iteration 12,757 to produce the final results shown below;

The appendix shows the programming code and manual calculation confirming the program output.

The results for the six (6) basic variables are:

$y_1 = 0.0476$, $y_2 = 0.0220$, $y_3 = 0.0565$, $y_4 = 0.0254$, $y_5 = 0.0003$, $y_6 = 0.0390$

Using the transformation $x_i = 101 y_i$, the values of the variables for the main Linear Programming problem are calculated as:

$$x_1 = 101 y_1 = 101(0.0476) = 4.8076$$

$$x_2 = 101 y_2 = 101(0.0220) = 2.2220$$

$$x_3 = 101 y_3 = 101(0.0565) = 5.7065$$

$$x_4 = 101 y_4 = 101(0.0254) = 2.5654$$

$$x_5 = 101 y_5 = 101(0.0003) = 0.0303$$

$$x_6 = 101 y_6 = 101(0.0390) = 3.9390$$

The optimal objective function value is

$$Z = 0.3622x_1 + 0.3192x_2 + 0.3464x_3 + 0.2852x_4 + 0.1560x_5 + 0.2025x_6$$

Thus

$$Z = 0.3622(4.8076) + 0.3192(2.2220) + 0.3464(5.7065) + 0.2852(2.5654) + 0.1560(0.0303) + 0.2025(3.9390)$$

or $Z = 5.9613$

4. DISCUSSION

Table 3 shows the table of fund allocations to be made by the management board of Capital bank as well as bad debt amount.

Using the above allocations, Capital Rural Bank Ltd. could realize a turn-over of **GH¢5,961,300.00** on loans as against **GH¢ 3,653,570.00** made in the previous year. This represents 61.3% increase in turn-over.

The Pearson correlation coefficient between the allocated amount and the bad debt amount was r = 0.026, which shows very low correlation between the allocated amount and the probability of bad debt. This shows that the allocations were influenced more by policy than the probability of bad debt. Agricultural loan, with the highest bad debt probability, was allocated the least amount of 30,300 while salary loan with the next bad debt probability of 0.10 obtained 5,706,500.00. This is because 60% of the total loan funds were allocated to salary, funeral, and commercial loans while only 15% of the total funds were to be used for agricultural and funeral loans. Because funeral loan shared in both the 60% and 15% allocations, it got a larger share from the 60% allocation and squeezed the agricultural loan into smaller share of the joint 15% policy allocation. Why should the allocation be influenced by the policy? This shows that Banks with weak policy always suffer and cannot maximize their profit on loans hence policy should be backed by scientific research before it is put in use.

Table 3. Summary of results for the allocation of funds

Variable	Loan Type	Probability of bad debt	Amount to be allocated (GH¢)	Bad debt amount (GHc)
x_1	Commercial Loan	0.020	4,807,600.00	96,152
x_2	Funeral Loan	0.030	2,222,000.00	66,660
x_3	Salary Loan	0.010	5,706,500.00	57,065
x_4	Susu Loan	0.055	2,565,400.00	141,097
x_5	Agricultural Loan	0.150	30,300.00	4,545
x_6	Housing Loan	0.075	3,939,000.00	295,425

5. CONCLUSION

Optimizing the disbursement of funds available for loans from Capital Rural Bank Limited would result in the appropriate allocation of funds to their customers. The disbursement is as follows: Commercial loan = GH¢4,807,600.00, Funeral loan = GH¢2,222,000.00, Salary loan = GH¢5,706,500.00, Susu loan = GH¢2,565,400.00, Agricultural loan = GH¢30,300.00 and Housing loan = GH¢3,939,000.00. The result shows that the bank would be able to make a maximum turn-over of **GH¢5,961,300.00** on loans as against **GH¢ 3,653,570.00** made in the previous year. This represents 61.3% increase in turnover. It has also been determined that fund allocation was influenced mostly by policy and not the probability of bad debt. However the bank should include efficient loan recovery methodology in their policy so that the limit of 4.5% total bad debt ratio will not be exceeded if not reduced. To reduce bad debts, effective monitoring of loan facilities through field visits and on regular basis and ensure that loans are in compliance with the terms and conditions of the facility, and identify potential problems for action to be taken. This would pre vent diversion of funds into business ventures other than the agreed purposes and also help loan officers assist customers who are facing some business management problems such as improper records keeping, and overtrading that affect their business operations.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Abbas TE, Majid S. Ank Loan Portfolio, an MOLP Based Modeling. International Journal of Finance and Economics. 2011;25:28–41.
2. Booth GG. Programming bank portfolios under uncertainty. Journal of Bank Research. 1972;3(4):28-40.
3. Cohen and Hamer. Linear programming and optimal Bank asset management decision. Journal of Finance. 1972;22(2): 147-165.
4. Klaassen P. Financial asset-pricing theory. Management Science-pubsonline.informs.org; 1998.
5. Dantzig GB. Reminiscences about the origins of linear programming. Mathematical Programming (R.W. Cottle, M.L. Kelmanson, B. Korte, eds.), Proceedings of the International Congress on Mathematical Programing, Rio de Janeiro. 1984;105-112.
6. Bazara (1984) and Hamdy (1992). Karmarkar's Projective Algorithm.
7. Karmarkar N. A new polynomial-time algorithm for linear programming, Combinatorica. 1984;4:373–395.
8. Wikimedia Foundation Inc. Karmarkar's Algorithm. Available:http://en.wikipedia.org/wiki/Karmarkar% 27s_algorithm (Accessed January, 2018)
9. Quintana VH. Interior-point methods and their applications; 2000.
10. Ferris and Philpott. The performance of Karmarkar's algorithm. Journal of the Operations Research Society; 1988.
11. Kojima M. A primal-dual interior point algorithm for linear programming; 1989.
12. Todd MJ. The affine-scalling direction for linear programming is a limit of projection scalling directions. Cornell University, Upson Hall Ithaca, New York; 1991.
13. Monteiro RDC. Interior Path following primal-dual algorithms; 1989.
14. Bayer and Lagarias. Affine and projective Scaling Trajectories. Brealey RA, Hodges SD. Playing with portfolios. The Journal of Finance, 1972;30(1):125-134.
15. Portfolios - An empirical analysis. Journal of Finance. 25(3):639-649.
16. Dhamija AK. Karmarkar's algorithm for linear programming problem; 2012. www.slideshare.net/akdhamija/karmarkars-algorithm-for-linear-programming

APPENDICES

APPENDIX A: Converting the LP Model of the Loan Optimization Problem Formulation into Karmarkar's Standard Form

The Standard Linear Programming form is as shown below:

Maximize

$$Z = 0.3622\ x_1 + 0.3192\ x_2 + 0.3464\ x_3 + 0.2852\ x_4 + 0.1560\ x_5 + 0.2025\ x_6$$

Subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 20$$
$$x_1 + x_2 + x_3 \leq 12$$
$$-0.5x_2 - 0.5x_3 + x_6 \leq 0$$
$$-0.4x_1 + x_4 + x_5 - 0.4x_6 \leq 0$$
$$x_2 + x_5 \leq 3$$
$$-0.025\ x_1 - 0.015\ x_2 - 0.035\ x_3 + 0.010\ x_4 + 0.105\ x_5 + 0.030\ x_6 \leq 0$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

- Writing the dual of the given primal problem:

Minimize $20\ w_1 + 12\ w_2 + 3w_5$

Subject to :

$$w_1 + w_2 - 0.4w_4 - 0.025\,w_6 \geq 0.3622$$
$$w_1 + w_2 - 0.5w_3 + w_5 - 0.015\,w_6 \geq 0.3192$$
$$w_1 + w_2 - 0.5w_3 - 0.035\,w_6 \geq 0.3464$$
$$w_1 + w_4 + 0.010\,w_6 \geq 0.2852$$
$$w_1 + w_4 + w_5 + 0.105\,w_6 \geq 0.1560$$
$$w_1 + w_3 - 0.4w_4 + 0.030\,w_6 \geq 0.2025$$
$$w_1, w_2, w_3, w_4, w_5, w_6 \geq 0$$

- Introduction of slack and surplus variables to the constraints of the primal and the dual respectively and then combine them

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 20$$
$$x_1 + x_2 + x_3 + x_8 = 12$$
$$-0.5x_2 - 0.5x_3 + x_6 + x_9 = 0$$
$$-0.4x_1 + x_4 + x_5 - 0.4x_6 + x_{10} = 0$$
$$x_2 + x_5 + x_{11} = 3$$
$$-0.025\,x_1 - 0.015\,x_2 - 0.035\,x_3 + 0.010\,x_4 + 0.105\,x_5 + 0.030\,x_6 + x_{12} = 0$$

- Addition of bounding constraint with slack variable ***s*** :

$$\sum_{i=1}^{12} x_i + \sum_{i=1}^{12} w_i + s = k$$
$$k = 100$$
$$\Rightarrow \sum_{i=1}^{12} x_i + \sum_{i=1}^{12} w_i + s = 100$$

$$w_1 + w_2 - 0.4w_4 - 0.025\,w_6 - w_7 = 0.3622$$
$$w_1 + w_2 - 0.5w_3 + w_5 - 0.015\,w_6 - w_8 = 0.3192$$
$$w_1 + w_2 - 0.5w_3 - 0.035\,w_6 - w_9 = 0.3464$$
$$w_1 + w_4 + 0.010\,w_6 - w_{10} = 0.2852$$
$$w_1 + w_4 + w_5 + 0.105\,w_6 - w_{11} = 0.1560$$
$$w_1 + w_3 - 0.4w_4 + 0.030\,w_6 - w_{12} = 0.2025$$
$$0.3622\,x_1 + 0.3192\,x_2 + 0.3464\,x_3 + 0.2852\,x_4 + 0.1560\,x_5 + 0.2025\,x_6 = 20\,w_1 + 12\,w_2 + 3w_5$$
$$w_i \geq 0, x_i \geq 0 \quad i = 1,2,3,\ldots, 12$$

- Homogenized equivalent system with dummy variable ***d*** :

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 - 20\,d = 0$$
$$x_1 + x_2 + x_3 + x_8 - 12\,d = 0$$
$$-0.5x_2 - 0.5x_3 + x_6 + x_9 = 0$$
$$-0.4x_1 + x_4 + x_5 - 0.4x_6 + x_{10} = 0$$
$$x_2 + x_5 + x_{11} - 3d = 0$$
$$-0.025\,x_1 - 0.015\,x_2 - 0.035\,x_3 + 0.010\,x_4 + 0.105\,x_5 + 0.030\,x_6 + x_{12} = 0$$
$$w_1 + w_2 - 0.4w_4 - 0.025\,w_6 - w_7 - 0.3622\,d = 0$$
$$w_1 + w_2 - 0.5w_3 + w_5 - 0.015\,w_6 - w_8 - 0.3192\,d = 0$$
$$w_1 + w_2 - 0.5w_3 - 0.035\,w_6 - w_9 - 0.3464\,d = 0$$
$$w_1 + w_4 + 0.010\,w_6 - w_{10} - 0.2852\,d = 0$$
$$w_1 + w_4 + w_5 + 0.105\,w_6 - w_{11} - 0.1560\,d = 0$$
$$w_1 + w_3 - 0.4w_4 + 0.030\,w_6 - w_{12} - 0.2025\,d = 0$$
$$0.3622\,x_1 + 0.3192\,x_2 + 0.3464\,x_3 + 0.2852\,x_4 + 0.1560\,x_5 + 0.2025\,x_6 - 20\,w_1 - 12\,w_2 - 3w_5 = 0$$

$$\sum_{i=1}^{12} x_i + \sum_{i=1}^{12} w_i + s - 100\,d = 0$$
$$\sum_{i=1}^{12} x_i + \sum_{i=1}^{12} w_i + s + d = 101$$
$$w_i \geq 0, x_i \geq 0 \quad i = 1,2,3,\ldots, 12$$

- Introduction of transformations:

$$x_i = (k+1)\,y_i \quad \Rightarrow \; x_i = 101\,y_i \quad i = 1,2,3,\ldots,12$$
$$w_i = (k+1)\,y_{12+i} \quad \Rightarrow \; w_i = 101\,y_{12+i} \quad i = 1,2,3,\ldots, 12$$
$$s = (k+1)\,y_{25} \quad \Rightarrow \; s = 101\,y_{25}$$
$$d = (k+1)\,y_{26} \quad \Rightarrow \; d = 101\,y_{26}$$

- Using the above transformations, the system becomes:

$$101(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 - 20y_{26}) = 0$$
$$101\,(y_1 + y_2 + y_3 + y_8 - 12\,y_{26}) = 0$$
$$101\,(-0.5y_2 - 0.5y_3 + y_6 + y_9) = 0$$
$$101\,(-0.4y_1 + y_4 + y_5 - 0.4y_6 + y_{10}) = 0$$
$$101\,(y_2 + y_5 + y_{11} - 3y_{26}) = 0$$
$$101(-0.025y_1 - 0.015y_2 - 0.035y_3 + 0.010y_4 + 0.105y_5 + 0.030y_6 + y_{12}) = 0$$

$101(y_{13} + y_{14} - 0.4y_{16} - 0.025y_{18} - y_{19} - 0.3622y_{26}) = 0$

$101(y_{13} + y_{14} - 0.5y_{15} + y_{17} - 0.015y_{18} - y_{20} - 0.3192y_{26}) = 0$

$101(y_{13} + y_{14} - 0.5y_{15} - 0.035y_{18} - y_{21} - 0.3464y_{26}) = 0$

$101(y_{13} + y_{16} + 0.010y_{18} - y_{22} - 0.2852y_{26}) = 0$

$101(y_{13} + y_{16} + y_{17} + 0.105y_{18} - y_{23} - 0.1560y_{26}) = 0$

$101(y_{13} + y_{15} - 0.4y_{16} + 0.030y_{18} - y_{24} - 0.2025y_{26}) = 0$

$101(0.3622y_1 + 0.3192y_2 + 0.3464y_3 + 0.2852y_4 + 0.1560y_5 + 0.2025y_6 - 20y_{13} - 12y_{14} - 3y_{17}) = 0$

$$101\left(\sum_{i=1}^{12} y_i + \sum_{i=13}^{24} y_i + y_{25} - 100\, y_{26}\right) = 0$$
$$101\left(\sum_{i=1}^{12} y_i + \sum_{i=13}^{24} y_i + y_{25} + y_{26}\right) = 101$$

- The system is further simplified to become:

$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 - 20y_{26} = 0$ $y_1 + y_2 + y_3 + y_8 - 12y_{26} = 0$
$-0.5y_2 - 0.5y_3 + y_6 + y_9 = 0$
$-0.4y_1 + y_4 + y_5 - 0.4y_6 + y_{10} = 0$
$y_2 + y_5 + y_{11} - 3y_{26} = 0$
$-0.025y_1 - 0.015y_2 - 0.035y_3 + 0.010y_4 + 0.105y_5 + 0.030y_6 + y_{12} = 0$
$y_{13} + y_{14} - 0.4y_{16} - 0.025y_{18} - y_{19} - 0.3622y_{26} = 0$

$y_{13} + y_{14} - 0.5y_{15} + y_{17} - 0.015y_{18} - y_{20} - 0.3192y_{26} = 0$

$y_{13} + y_{14} - 0.5y_{15} - 0.035y_{18} - y_{21} - 0.3464y_{26} = 0$

$y_{13} + y_{16} + 0.010y_{18} - y_{22} - 0.2852y_{26} = 0$

$y_{13} + y_{16} + y_{17} + 0.105y_{18} - y_{23} - 0.1560y_{26} = 0$

$y_{13} + y_{15} - 0.4y_{16} + 0.030y_{18} - y_{24} - 0.2025y_{26} = 0$

$0.3622y_1 + 0.3192y_2 + 0.3464y_3 + 0.2852y_4 + 0.1560y_5 + 0.2025y_6 - 20y_{13} - 12y_{14} - 3y_{17} = 0$

$$\sum_{i=1}^{25} y_i - 100\, y_{26} = 0$$
$$\sum_{i=1}^{26} y_i = 1$$
$y_i \geq 0$, $i = 1,2,3,\ldots, 26$

An artificial variable y_{27} is introduced and the Karmarkar's form is :

Minimize $M = y_{27}$

Subject to:

$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 - 20y_{26} + 13y_{27} = 0$ $y_1 + y_2 + y_3 + y_8 - 12y_{26} + 8y_{27} = 0$
$-0.5y_2 - 0.5y_3 + y_6 + y_9 - y_{27} = 0$

$$-0.4y_1 + y_4 + y_5 - 0.4y_6 + y_{10} - 2.2y_{27} = 0$$
$$y_2 + y_5 + y_{11} - 3y_{26} = 0$$
$$-0.025y_1 - 0.015y_2 - 0.035y_3 + 0.010y_4 + 0.105y_5 + 0.030y_6 + y_{12} - 1.07y_{27} = 0$$
$$y_{13} + y_{14} - 0.4y_{16} - 0.025y_{18} - y_{19} - 0.3622y_{26} - 0.2128y_{27} = 0$$
$$y_{13} + y_{14} - 0.5y_{15} + y_{17} - 0.015y_{18} - y_{20} - 0.3192y_{26} - 1.1658y_{27} = 0$$
$$y_{13} + y_{14} - 0.5y_{15} - 0.035y_{18} - y_{21} - 0.3464y_{26} - 0.1186y_{27} = 0$$
$$y_{13} + y_{16} + 0.010y_{18} - y_{22} - 0.2852y_{26} - 0.7248y_{27} = 0$$
$$y_{13} + y_{16} + y_{17} + 0.105y_{18} - y_{23} - 0.1560y_{26} - 1.949y_{27} = 0$$
$$y_{13} + y_{15} - 0.4y_{16} + 0.030y_{18} - y_{24} - 0.2025y_{26} + 1.5725y_{27} = 0$$
$$0.3622y_1 + 0.3192y_2 + 0.3464y_3 + 0.2852y_4 + 0.1560y_5 + 0.2025y_6 - 20y_{13} - 12y_{14} - 3y_{17} + 33.3285y_{27} = 0$$
$$\sum_{i=1}^{25} y_i - 100y_{26} + 75y_{27} = 0$$
$$\sum_{i=1}^{27} y_i = 1$$
$$y_i \geq 0, \quad i = 1,2,3,\ldots,27$$

APPENDIX B: MATLAB Code to Solve the Karmarkar's Standard form in Appendix B

The following Matlab Code for Karmarkar's Algorithm was written by the authors and was used to solve the Karmarkar Formulation for the Loan Optimization Problem

```
% A is m x n matrix
% C is a column vector
% E is a row vector with 1's as its entries
% k is the number of iterations
% tol is the tolerance
A=input('Enter the matrix A:')
C=input('Enter  C:')
E=input('Enter  E:')
mn =size(A)
m=mn(1)
n=mn(2)
V1= ([E]/n)'
I=eye (n)
r=1/sqrt(n*(n-1))
α= (n-1)/(3*n)
V= ([E]/n)'
tol=10^(-15)
k=0
while(C'*V>tol)
D=diag(V)
T=A*D
P= [T; E]
Q=C'*D
R= (I-P'/(P*P' )*P)*Q'
RN=norm (R)
Y=V1-(r*α)*(R/RN)
V= (D*Y)/(E*D*Y)
M=C'*V
k=k+1
end
```


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