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Formal System of Categorical Syllogistic Logic Based on the Syllogism *AEE-4*

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Abstract

Adopting a different method from the previous scholars, this article deduces the remaining 23 valid syllogisms just taking the syllogism *AEE-4* as the basic axiom. The basic idea of this study is as follows: firstly, make full use of the trichotomy structure of categorical propositions to formalize categorical syllogisms. Then, taking advantage of the deductive rules in classical propositional logic and the basic facts in the generalized quantifier theory, we deduce the remaining 23 valid categorical syllogisms by taking just one syllogism (that is, *AEE-4*) as the basic axiom. This article not only reveals the reducible relations between the syllogism *AEE-4* and the other 23 valid syllogisms, but also establishes a concise formal axiomatic system for categorical syllogistic logic. We hope that the results and methods will provide a good mathematical paradigm for studying other kinds of syllogistic logics, and that the project will appeal to specialists in logic, linguistic semantics, computational semantics, cognitive science and artificial intelligence.

Keywords

Aristotelian Quantifiers, Symmetry, Categorical Syllogisms, Reduction

1. Introduction

Syllogisms are important forms of reasoning in natural language and logic from Aristotle onwards. There are various syllogisms in natural language, such as categorical syllogisms (Moss, 2008), modal syllogisms (Zhang, 2020a, 2020b), generalized syllogisms (Murinová & Novák, 2012), relational syllogisms (Pratt-Hartmann, 2009, 2014), syllogisms with adjectives (Moss, 2011), and so on. Among them, categorical syllogisms have a long history of research and are widely used in human reasoning (Chen, 2000). Categorical syllogisms involve sentences of the following four forms: *all* xs are y, *no* xs are y, *some* xs are y, and *not all* xs are y.

This article focuses on the time-honored categorical syllogistic logic which has been discussed from different perspectives since Aristotle, for example by Łukasiewicz (1957), Corcoran (1972), van Benthem (1984), Westerståhl (1989), Martin (1997), and Zhang (2016, 2020a, 2020b, 2021), and so on. The reason why categorical syllogistic logic is widely studied is that it is a common form of reasoning in natural languages.

It is well known that merely 24 of 256 types of categorical syllogisms are valid (Chen, 2000). When deriving all of the other valid syllogisms, at least two valid syllogisms were used as basic axioms in previous studies, for example by Łukasiewicz (1957), Cai (1988), Zhang (2016, 2018) and Zhou et al. (2018). Adopting a different approach from the previous scholars, this article deduces the remaining 23 valid syllogisms taking just one syllogism (that is, *AEE-4*) as the basic axiom.

2. Relevant Preliminary Knowledge

In this article, Q represents one of the four Aristotelian quantifiers (that is, *all*, *some*, *no*, *not all*), x, y and z represent lexical variables, and D indicates the domain of lexical variables. In order to express concisely, D is omitted in contexts or without ambiguity.

An Aristotelian syllogism contains three categorical propositions, two of which are premises and one is conclusion. Categorical propositions include the following four types of propositions: A, E, I and O. The proposition A is a universal affirmative proposition, which means that all xs are y and can be formalized as $all(x, y)$. The proposition E is a universal negative proposition, which means that no xs are y and can be denoted as $no(x, y)$. The proposition I is a particular affirmative proposition, which means that some xs are y and can be formalized as $some(x, y)$. The proposition O is a particular negative proposition, which means that not all xs are y and can be symbolized as $not\ all(x, y)$. The definition of figures of syllogisms is as usual. The syllogism *AEE-4* indicates the fourth figure of a syllogism which its major premise, minor premise and conclusion are respectively the proposition A, E and E. And then the syllogism *AEE-4* is denoted as $all(y, z) \wedge no(z, x) \rightarrow no(x, y)$. Other formal representations are similar.

3. The Structure of Axiomatic System of Categorical Syllogisms

This formalized axiom system is structured on the basis of the following four parts: initial symbols, formation rules for well-formed formulas, axioms, and rules of deduction.

3.1. Primitive Symbols

(1) lexical variables: x, y, z

(2) quantifier: *all*

(3) unary negative operator: $\neg$

(4) binary conjunction operator: $\wedge$

(5) binary implication operator: $\rightarrow$

(6) brackets: (,)

3.2. Formation Rules

(1) If Q is a quantifier, x and y are lexical variables, then $Q(x, y)$ is a well-formed formula.

(2) If p is a well-formed formula, then $\neg p$ is well-formed formula.

(3) If p and q are well-formed formulas, then $p \wedge q$ and $p \rightarrow q$ are well-formed formulas.

(4) Only the formulas obtained by the above three rules are well-formed formulas.

3.3. Basic Axioms

(1) A1: if p is a valid formula in classical propositional logic, then $\vdash p$.

(2) A2: $\vdash all(y, z) \wedge no(z, x) \rightarrow no(x, y)$ (that is, the syllogism *AEE-4*).

3.4. Rules of Deduction

The following deductive rules in classical propositional logic (c.f. Hamilton (1978)) are also applicable in categorical syllogistic logic. In the following rules, p, q, r and s are well-formed formulas. $\vdash p$ means that p is provable. The other notations are similar. And the replacement rule is used by default in this article.

(1) Rule 1 (antecedent interchange): From $\vdash(p \wedge q \rightarrow r)$ infer $\vdash(q \wedge p \rightarrow r)$.

(2) Rule 2 (subsequent weakening): From $\vdash(p \wedge q \rightarrow r)$ and $\vdash(r \rightarrow s)$ infer $\vdash(p \wedge q \rightarrow s)$.

(3) Rule 3 (anti-syllogism): From $\vdash(p \wedge q \rightarrow r)$ infer $\vdash(\neg r \wedge p \rightarrow \neg q)$.

3.5. Relevant Definitions

(1) Definition of connective $\leftrightarrow$: $(p \leftrightarrow q) =_{def} (p \rightarrow q) \wedge (q \leftarrow p)$

(2) Definition of inner negative quantifier: $(Q\neg)(x, y) =_{def} Q(x, D-y)$

(3) Definition of outer negative quantifier: $(\neg Q)(x, y) =_{def}$ It is not that $Q(x, y)$

(4) Definition of dual quantifier: $\neg Q\neg(x, y) =_{def}$ It is not that $Q(x, D-y)$

The categorical syllogisms characterize the semantic and inferential properties of the four Aristotelian quantifiers (that is, *all*, *no*, *some* and *not all*). The reason why this article only takes one Aristotelian quantifier (i.e., *all*) as the initial quantifier is that the other three Aristotelian quantifiers can be defined by this one. More specifically, $no =_{def} all\neg$, $not\ all =_{def} \neg all$, and $some =_{def} \neg all\neg$ by the above definitions.

3.6. Relevant Facts

The following four facts are the basic facts in the generalized quantifier theory (c.f. Peters & Westerståhl (2006), and Zhang (2014)), which can be easily proved by using the above definitions, axioms, and rules of deduction.

Fact 1 (inner negation):

(1) $\vdash all(x, y) \leftrightarrow no\neg(x, y)$;   (2) $\vdash \text{no}(x, y) \leftrightarrow \text{all}\neg(x, y)$;

(3) $\vdash some(x, y) \leftrightarrow not\ all\neg(x, y)$; (4) $\vdash not\ all(x, y) \leftrightarrow some\neg(x, y)$.

Fact 2 (outer negation):

(1) $\vdash \neg not\ all(x, y) \leftrightarrow all(x, y)$; (2) $\vdash \neg all(x, y) \leftrightarrow not\ all(x, y)$;

(3) $\vdash \neg no(x, y) \leftrightarrow some(x, y)$; (4) $\vdash \neg some(x, y) \leftrightarrow no(x, y)$.

Fact 3 (symmetry):

(1) symmetry of *some*: $\vdash some(x, y) \leftrightarrow some(y, x)$; (2) symmetry of *no*: $\vdash no(x, y) \leftrightarrow no(y, x)$.

Fact 4 (assertoric subalternations):

(1) $\vdash all(x, y) \rightarrow some(x, y)$; (2) $\vdash no(x, y) \rightarrow not\ all(x, y)$.

4. The Reduction from the Syllogism *AEE-4* to the Remaining 23 Valid Syllogisms

In the following theorem 1, *AEE-4*⇒*AEE-2* means that the validity of the syllogism *AEE-2* can be deduced from the validity of the syllogism *AEE-4*. In other words, the two syllogisms are reducible. Other notations are similar.

Theorem 1: The remaining 23 valid syllogisms can be deduced merely from the syllogism *AEE-4*. According to the order and steps of the proof, the following can be obtained:

(1) *AEE-4*⇒*AEE-2*

(2) *AEE-4*⇒*AEE-2*⇒*EAE-2*

(3) *AEE-4*⇒*EAE-1*

(4) *AEE-4*⇒*AEO-4*

(5) *AEE-4*⇒*AEO-4*⇒*AEO-2*

(6) *AEE-4*⇒*AEE-2*⇒*EAE-2*⇒*EAO-2*

(7) *AEE-4*⇒*EAE-1*⇒*EAO-1*

(8) *AEE-4*⇒*AEE-2*⇒*AII-1*

(9) *AEE-4*⇒*AEE-2*⇒*AII-1*⇒*AII-3*

(10) *AEE-4*⇒*AEE-2*⇒*AII-1*⇒*AII-3*⇒*IAI-3*

(11) *AEE-4*⇒*AEE-2*⇒*AII-1*⇒*AII-3*⇒*IAI-3*⇒*IAI-4*

(12) *AEE-4*⇒*AEO-4*⇒*AEO-2*⇒*EAO-3*

(13) *AEE-4*⇒*AEO-4*⇒*AEO-2*⇒*EAO-3*⇒*EAO-4*

(14) *AEE-4*⇒*AEO-4*⇒*AEO-2*⇒*AAI-3*

(15) *AEE-4*⇒*EAE-1*⇒*AAA-1*

(16) *AEE-4*⇒*EAE-1*⇒*AAA-1*⇒*AAI-1*

(17) *AEE-4*⇒*EAE-1*⇒*AAA-1*⇒*AAI-1*⇒*AAI-4*

(18) *AEE-4*⇒*EAE-1*⇒*AAA-1*⇒*OAO-3*

(19) *AEE-4*⇒*EAE-1*⇒*AAA-1*⇒*OAO-3*⇒*AOO-2*

(20) *AEE-4*⇒*AEE-2*⇒*AII-1*⇒*EIO-1*

(21) *AEE-4*⇒*AEE-2*⇒*AII-1*⇒*EIO-1*⇒*EIO-3*

(22) *AEE-4*⇒*AEE-2*⇒*AII-1*⇒*EIO-1*⇒*EIO-3*⇒*EIO-4*

(23) *AEE-4*⇒*AEE-2*⇒*AII-1*⇒*EIO-1*⇒*EIO-3*⇒*EIO-4*⇒*EIO-2*

Proof:

[1] $\vdash all(y, z) \wedge no(z, x) \rightarrow no(x, y)$ (i.e. *AEE-4*, basic axiom A2)

[2] $\vdash no(z, x)\leftrightarrow no(x, z)$	(by (2) of Fact 3)
[3] $\vdash all(y, z)\wedge no(x, z)\rightarrow no(x, y)$	(i.e. *AEE-2*, by [1] and [2])
[4] $\vdash all(y, z)\wedge no(x, z)\rightarrow no(y, x)$	(i.e. *EAE-2*, by [3] and (2) of Fact 3)
[5] $\vdash all(y, z)\wedge no(z, x)\rightarrow no(y, x)$	(i.e. *EAE-1*, by [1] and (2) of Fact 3)
[6] $\vdash no(x, y)\rightarrow not\ all(x, y)$	(by (2) of Fact 4)
[7] $\vdash all(y, z)\wedge no(z, x)\rightarrow not\ all(x, y)$	(i.e. *AEO-4*, by [1], [6] and Rule 2
[8] $\vdash all(y, z)\wedge no(x, z)\rightarrow not\ all(x, y)$	(i.e. *AEO-2*, by [2] and [7])
[9] $\vdash all(y, z)\wedge no(x, z)\rightarrow not\ all(y, x)$	(i.e. *EAO-2*, by [4], (2) of Fact 4 and Rule 2)
[10] $\vdash all(y, z)\wedge no(z, x)\rightarrow not\ all(y, x)$	(i.e. *EAO-1*, by [2] and [9])
[11] $\vdash\neg no(x, y)\wedge all(y, z)\rightarrow\neg no(x, z)$	(by [3] and Rule 3)
[12] $\vdash some(x, y)\wedge all(y, z)\rightarrow some(x, z)$	(i.e. *AII-1*, by [11] and (3) of Fact 2)
[13] $\vdash some(y, x)\wedge all(y, z)\rightarrow some(x, z)$	(i.e. *AII-3*, by [12] and (1) of Fact 3)
[14] $\vdash some(y, x)\wedge all(y, z)\rightarrow some(z, x)$	(i.e. *AII-3*, by [13] and (1) of Fact 3)
[15] $\vdash some(x, y)\wedge all(y, z)\rightarrow some(z, x)$	(i.e. *IAI-4*, by [14] and (1) of Fact 3)
[16] $\vdash\neg not\ all(x, y)\wedge no(x, z)\rightarrow\neg all(y, z)$	(by [8], Rule 1 and Rule 3)
[17] $\vdash no(x, z)\wedge all(x, y)\rightarrow not\ all(y, z)$	(i.e. *EAO-3*, by [16], (1) and (2) of Fact 2, and Rule 1)
[18] $\vdash no(z, x)\wedge all(x, y)\rightarrow not\ all(y, z)$	(i.e. *EAO-4*, by [2] and [17])
[19] $\vdash\neg not\ all(y, x)\wedge all(y, z)\rightarrow\neg no(x, z)$	(by [9] and Rule 3)
[20] $\vdash all(y, x)\wedge all(y, z)\rightarrow some(x, z)$	(i.e. *AAI-3*, by [19], (1) and (3) of Fact 2)
[21] $\vdash all(y, z)\wedge all\neg(z, x)\rightarrow all\neg(y, x)$	(by [5] and (2) of Fact 1)
[22] $\vdash all(y, z)\wedge all(z, D-x)\rightarrow all(y, D-x)$	(by [21] and (2) of Definition (3.5))
[23] $\vdash all(y, z)\wedge all(z, x)\rightarrow all(y, x)$	(i.e. *AAA-1*, by [22])
[24] $\vdash all(y, z)\wedge all(z, x)\rightarrow some(y, x)$	(i.e. *AAI-1*, by [23], (1) of Fact 4 and Rule 2)
[25] $\vdash all(y, z)\wedge all(z, x)\rightarrow some(x, y)$	(i.e. *AAI-4*, by [24] and (1) of Fact 3)
[26] $\vdash\neg all(y, x)\wedge all(y, z)\rightarrow\neg all(z, x)$	(by [23] and Rule 3)
[27] $\vdash not\ all(y, x)\wedge all(y, z)\rightarrow not\ all(z, x)$	(i.e. *OAO-3*, by [26] and (2) of Fact 2)
[28] $\vdash\neg not\ all(z, x)\wedge not\ all(y, x)\rightarrow\neg all(y, z)$	(by [27] and Rule 3)
[29] $\vdash all(z, x)\wedge not\ all(y, x)\rightarrow not\ all(y, z)$	(i.e. *AOO-2*, by [28], (1) and (2) of Fact 2)
[30] $\vdash some(x, y)\wedge no\neg(y, z)\rightarrow not\ all\neg(x, z)$	(by [12], (1) and (3) of Fact 1)

[31] $\vdash no(y, D-z) \wedge some(x, y) \rightarrow not\ all(x, D-z)$	(by [30] and (2) of Definition (3.5))
[32] $\vdash no(y, z) \wedge some(x, y) \rightarrow not\ all(x, z)$	(i.e. *EIO-1*, by [31])
[33] $\vdash no(y, z) \wedge some(y, x) \rightarrow not\ all(x, z)$	(i.e. *EIO-3*, by [32] and (1) of Fact 3)
[34] $\vdash no(z, y) \wedge some(y, x) \rightarrow not\ all(x, z)$	(i.e. *EIO-4*, by [33] and (2) of Fact 3)
[35] $\vdash no(z, y) \wedge some(x, y) \rightarrow not\ all(x, z)$	(i.e. *EIO-2*, by [34] and (1) of Fact 3)

5. Conclusion and Future Work

The basic idea of this study is as follows: Firstly, make full use of the trichotomy structure of categorical proposition to formalize categorical syllogisms. Then, taking advantage of the deductive rules in classical propositional logic and the basic facts in the generalized quantifier theory, we can deduce the other 23 valid categorical syllogisms by taking just one syllogism (that is, *AEE-4*) as the basic axiom. This article not only reveals the reducible relations between the syllogism *AEE-4* and the other 23 valid syllogisms, but also establishes a concise formal axiomatic system for categorical syllogistic logic. The research methods and results are concise, clear, and enlightening.

The basic steps for computer to process a statement in natural language are as follows: first, formalize the statement; then give the algorithm of its formal expression; finally, compile the program according to the algorithm. In other words, formalizing sentences in natural language is the first step of natural language information processing. This paper makes a formal study of categorical syllogisms from the perspective of mathematical structuralism and generalized quantifier theory. This study not only provides a universal mathematical paradigm for studying other kinds of syllogisms, but also provides theoretical support for natural language information processing, knowledge representation and knowledge reasoning in computer science.

How to integrate the research results of generalized quantifier theory and categorical syllogistic logic to further improve their role in the intersection of logic, natural language processing and computer science, and how to make the best of the spillover effects in theoretical research to deal with practical problems and promote computer context awareness and knowledge reasoning? These issues need to be explored in depth.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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