

## Research Article

# A Simple Method to Differentiate between Free-Carrier Recombination and Trapping Centers in the Bandgap of the p-Type Semiconductor

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In this research, the ranges of the localized states in which the recombination and the trapping rates of free carriers dominate the entire transition rates of free carriers in the bandgap of the p-type semiconductor are described. Applying the Shockley–Read–Hall model to a p-type material under a low injection level, the expressions for the recombination rates, the trapping rates, and the excess carrier lifetimes (recombination and trapping) were described as functions of the localized state energies. Next, the very important quantities called the excess carriers' trapping ratios were described as functions of the localized state energies. Variations of the magnitudes of the excess carriers' trapping ratios with the localized state energies enable us to categorize the localized states in the bandgap as the recombination, the trapping, the acceptor, and the donor levels. Effects of the majority and the minority carriers' trapping on the excess carrier lifetimes are also evaluated at different localized energy levels. The obtained results reveal that only excess minority trapping affects the excess carrier lifetimes, and excess majority carrier trapping has no effect.

## 1. Introduction

Carrier transition through the localized state of the indirect band semiconductor involves the capture of electrons from the conduction band by the localized state, followed by the recombination with holes in the valence band at the localized state or the reemission of electrons or holes back to their original band [1]. When electron-hole pairs recombine at the localized state, energy is released through phonon emission, and the localized state is called the recombination center [1, 2]. This type of recombination of electron-hole pairs is called Shockley–Read–Hall (SRH) recombination [1]. If the localized state captures free carriers temporarily and then reemits them back to their original band, the localized state is called a free-carrier trap [1–5]. These properties depend on the shallowness and the deepness of the localized state in the bandgap of the material [6–14]. Different localized states have different capture coefficients for free carriers. Different

measurement methods also give widely differing values of the free-carrier capture cross sections for the same material or device structure [5, 15]. Values for the free-carrier capture cross sections ranged from  $10^{-21}$  cm<sup>2</sup> to  $10^{-13}$  cm<sup>2</sup> which have been reported by several researchers in silicon-based devices and different materials [6–14, 16–19]. Dudeck and Kassing [10] determined the temperature dependence of the capture cross section of negatively charged gold centers in n-type silicon for holes in the temperature range of 55 to 340 K from the temperature-dependent breakdown voltage of gold-compensated PIN diodes showing a negative differential resistance region. Based on a simplified analysis of the kinetics of filling of the traps in the slow regime of filling, Stevenard et al. [13] presented a new technique of measuring capture cross sections in semiconductors. In his work on a new radio frequency-based pulse rise time method with a single non-contact direct measurement, Kamieniecki [14] also reported the effect of charge trapping on the transient behaviors of the

effective carrier lifetime in compound semiconductors. The investigations of Hangleiter [16] on the deep levels induced by transition metal impurities in silicon and those of Schroder [19] on various recombination mechanisms in silicon are also the most reliable sources on silicon.

Several researchers have tried to describe the effects of free-carrier trapping on the photoresponses of semiconductors analytically [1–5, 11, 15, 18, 20–22]. Macdonald and Cuevas described the validity of Shockley–Read–Hall statistics for modeling carrier lifetimes in crystalline silicon [15]. Roosbroeck [20] also derived fundamental differential equations that admit trapping under the unrestricted approximation of electrical neutrality in which the general transport equations hold without explicit reference to detailed trapping and recombination statistics. In his work on the description of nonradiative capture and recombination by multiphonon emission in GaAs and GaP, Henry and Lang [11] reported the exponential variation of capture cross section with temperature and activation energy. Schroder and Mitonneau et al. [5, 21] formulated the expression describing the variation of the free carriers' capture cross sections of the localized states with temperature and activation energies with respect to the valence and conduction bands. However, to our knowledge, there are no detailed statistical and numerical reports on the explicit ranges of the recombination centers, the excess hole, and electron trapping centers in the bandgap of semiconductors. No detailed analytical reports are also present on how the majority and the minority carrier trapping can affect the photoresponses of semiconductors.

This research starts its presentation with the description of the accumulation rates of the photogenerated carriers in the conduction band, the valence band, and the localized states in the bandgap of p-type silicon under the low injection level. Using the low injection level system makes the trapping ratios, all the excess carrier lifetimes, the normalized trapping, and emission rates of excess carriers to be independent of the illumination parameters and leaves only as functions of the thermal properties of the material. Macdonald et al. [23] also realized this fact by stating that photoluminescence-based measurements were found to offer some distinct advantages over traditional photoconductance-based techniques in determining recombination parameters from low-injection carrier lifetimes. Silicon (Si) is selected due to its indirect bandgap ranged from 1.17 eV at absolute zero to 1.12 eV at room temperature [5] and the large number of localized states identified in its bandgap [16–19]. The low injection level steady-state conditions of these accumulation rates give simplest expressions for the excess carrier densities in the conduction band, the valence band, and the localized state in terms of the thermal equilibrium majority carrier density, the thermal equilibrium minority carrier density, the localized state density, the activation energy of the localized state, and the illumination rate. This is followed by the descriptions of the excess electron trapping ratio  $\gamma_n$  given by the ratio of the density of the excess electrons in the localized state to that of the excess electrons in the conduction band and the excess hole trapping ratio  $\gamma_p$  which is the ratio of the density of excess holes in the localized state to that of the excess holes in the valence band. Using the curves of the localized state

energy dependences of  $\gamma_n$  and  $\gamma_p$ , one can clearly show the ranges of energies where the trapping, the recombination, the donor, and the acceptor effects become dominant. The excess carrier lifetimes corresponding to the recombination and the trapping of excess carriers at the localized states can be described by dividing the excess carrier densities at the respective bands and the localized states by the recombination and the trapping rates at the bands and the localized states.

The overall obtained results show that the SRH excess carrier recombination rate in p-type material dominates at a localized-state energy far below the midgap and above the Fermi energy level called the recombination center. The hole trapping and emission rate between the localized states and the valence band in p-type material also dominates the localized states of energy near the Fermi energy level. The electron trapping and emission rates between the localized states and the conduction band also dominate the localized states of energy between the recombination center and the donor level. It is also confirmed in this work that only excess minority trapping has a major effect on the performance of the semiconductor material, and majority carrier trapping has a minor effect.

## 2. Theory

Figure 1 shows the energy band structure of a semiconductor with one localized state. The density of the localized state  $N_T$  is given by the sum of the thermal equilibrium hole and electron occupancy,  $p_{0T}$  and  $n_{0T}$ , of a localized state [1]:

$$N_T = p_{0T} + n_{0T}. \quad (1)$$

The thermal equilibrium hole and electron occupancy,  $p_{0T}$  and  $n_{0T}$ , of a localized state are given in terms of the Fermi energy level,  $E_F$ , the localized state energy,  $E_T$ , and the density of the localized state,  $N_T$ , as follows [1, 2, 5]:

$$p_{0T} = \frac{N_T}{1 + \exp(E_F - E_T/k_B T)}, \quad (2)$$

$$n_{0T} = \frac{N_T}{1 + \exp(E_T - E_F/k_B T)},$$

where  $k_B$  is the Boltzmann constant and  $T$  is the temperature of the system. According to the Shockley–Read–Hall model, there are four transition processes of electrons and holes via the localized state described as the electron and hole capture rates ( $U_{cn}$  and  $U_{cp}$ ) and emission rates ( $U_{en}$  and  $U_{ep}$ ) by the localized states as shown in Figure 1 [1]. The thermal equilibrium electron and hole capture rates,  $U_{cn}^0$  and  $U_{cp}^0$ , are functions of the densities of the electron and hole in the conduction and valence bands,  $n_0$  and  $p_0$ , and the densities of empty and occupied localized states,  $p_{0T}$  and  $n_{0T}$ , respectively [1]:

$$U_{cn}^0 = C_{nT} n_0 p_{0T}, \quad (3)$$

$$U_{cp}^0 = C_{pT} p_0 n_{0T},$$

where  $C_{nT}$  and  $C_{pT}$  are called the electron and hole capture coefficients of the localized state described in terms of the

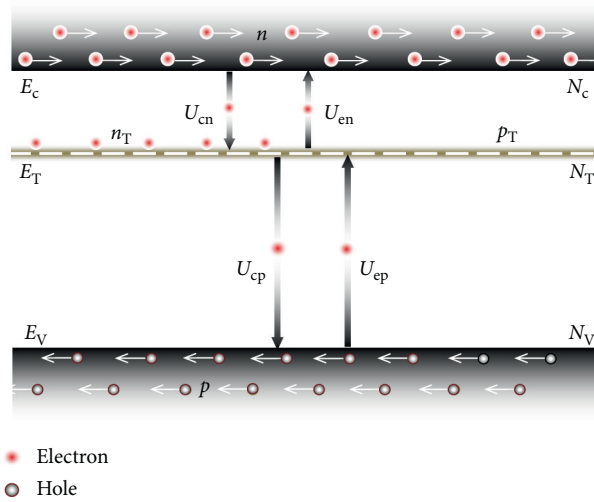


FIGURE 1: Energy band structure with a localized state.

capture cross-section areas of the localized state for free electrons and holes,  $\sigma_{nT}$  and  $\sigma_{pT}$ , and the mean thermal speeds of free electrons and holes,  $v_n$  and  $v_p$ , respectively, given by

$$\begin{aligned} C_{nT} &= \sigma_{nT} v_n, \\ C_{pT} &= \sigma_{pT} v_p. \end{aligned} \quad (4)$$

The mean thermal speeds of free electrons and holes,  $v_n$  and  $v_p$ , are also described in terms of the electron and hole density of states' effective masses in the conduction and valence bands,  $m_n^*$  and  $m_p^*$ , respectively, as follows:

$$\begin{aligned} v_n &= \sqrt{\frac{8k_B T}{\pi m_n^*}}, \\ v_p &= \sqrt{\frac{8k_B T}{\pi m_p^*}}. \end{aligned} \quad (5)$$

The carriers' capture coefficient of a center which is the product of the capture cross section and the thermal velocity of the carrier in the material can be thought of as the volume swept out per unit time by a charge carrier [1]. The thermal velocities of the carriers depend on the effective masses of the charge carriers in the material, whereas the free-carrier capture cross sections depend on the proximity of the center to the conduction and the valence bands. The capture cross-sectional areas of the localized state for free holes and electrons,  $\sigma_{pT}$  and  $\sigma_{nT}$ , are given as functions of temperature,  $T$ , and the activation energies of the localized states relative to the valence and conduction band edges,  $E_T - E_V$  and  $E_C - E_T$ , respectively [5, 11, 21]:

$$\begin{aligned} \sigma_{pT} &= \sigma_{\infty} \exp\left(-\frac{E_T - E_V}{k_B T}\right), \\ \sigma_{nT} &= \sigma_{\infty} \exp\left(-\frac{E_C - E_T}{k_B T}\right), \end{aligned} \quad (6)$$

where  $\sigma_{\infty}$  is the maximum capture cross section of the shallow localized states for free carriers with the value as

large as  $10^{-15} \text{cm}^2$  in most of the semiconductors that is independent of temperature and localized energy level. It can be shown from the above relations that the capture cross sections  $\sigma_{nT}$  and  $\sigma_{pT}$  are equal at the midgap, and the two capture coefficients  $C_{pT}$  and  $C_{nT}$  are also equal at the localized state of energy:

$$E_{cT} = \frac{E_V + E_C}{2} - \frac{k_B T}{4} \ln\left(\frac{m_p^*}{m_n^*}\right). \quad (7)$$

One can see from equation (7) that  $E_{cT}$  is located below the midgap in the p-type semiconductor ( $m_p^* > m_n^*$ ). The room temperature value of  $E_{cT}$  in the case of silicon is about 0.406 meV below the midgap. The emission rates  $U_{en}^0$  and  $U_{ep}^0$  also depend on  $n_{0T}$  and  $p_{0T}$  and the probabilities of electron and hole emissions,  $e_n$  and  $e_p$ , respectively [1]:

$$\begin{aligned} U_{en}^0 &= e_n n_{0T}, \\ U_{ep}^0 &= e_p p_{0T}. \end{aligned} \quad (8)$$

According to the principle of detailed balance, the rates of capture and emission for each carrier are equal. That is,

$$\begin{aligned} U_{cn}^0 &= U_{en}^0, \\ U_{cp}^0 &= U_{ep}^0. \end{aligned} \quad (9)$$

Taking this into account along with equations (3) and (8) yields

$$\begin{aligned} e_n &= C_{nT} n_1, \\ e_p &= C_{pT} p_1, \end{aligned} \quad (10)$$

where  $n_1$  and  $p_1$  are the concentrations of holes and electrons in the conduction and valence bands, respectively, for the case when the Fermi level,  $E_F$ , falls at  $E_T$  and are given by [1]

$$\begin{aligned} n_1 &= n_i \exp\left(\frac{E_T - E_i}{k_B T}\right), \\ p_1 &= n_i \exp\left(\frac{E_i - E_T}{k_B T}\right). \end{aligned} \quad (11)$$

The two concentrations  $n_1$  and  $p_1$  also satisfy the law of mass action  $n_1 p_1 = n_i^2$ . The thermal equilibrium emission rates  $U_{en}^0$  and  $U_{ep}^0$  in equation (8) hence take the form

$$\begin{aligned} U_{en}^0 &= C_{nT} n_1 n_{0T}, \\ U_{ep}^0 &= C_{pT} p_1 p_{0T}. \end{aligned} \quad (12)$$

If the equilibrium is now disturbed by illumination, the photogenerated electrons and holes are involved in the conduction processes in the conduction and the valence bands, respectively. The total free electron and hole densities in the conduction and valence band  $n$  and  $p$  become the sum of the densities of thermal electrons ( $n_0$ ) and holes ( $p_0$ ) and the photogenerated (excess) electrons ( $\delta n$ ) and holes ( $\delta p$ ), respectively:

$$\begin{aligned} n &= n_0 + \delta n, \\ p &= p_0 + \delta p. \end{aligned} \quad (13)$$

Since the localized state density  $N_T$  does not vary with illumination, equation (1) holds true for both the thermal and the nonthermal equilibrium conditions, while the electron and hole occupancy,  $n_T$  and  $p_T$ , of the localized state under illumination are changed by  $\delta n_T$  and  $\delta p_T$  relative to the thermal equilibrium occupancy,  $n_{0T}$  and  $p_{0T}$ , respectively:

$$\begin{aligned} N_T &= n_T + p_T \\ &= (n_{0T} + \delta n_T) + (p_{0T} + \delta p_T), \end{aligned} \quad (14)$$

where  $\delta n_T$  and  $\delta p_T$  are the densities of states filled by excess electrons and holes, respectively, during illumination. One can see from the equations (1) and (14) that

$$\delta n_T = -\delta p_T. \quad (15)$$

The negative sign (-) in equation (15) shows that the trapping of electrons by the localized state removes the same amount of  $p_T$  states. In a system with free carriers' trap, the net excess carrier density is shared in between the localized states and the bands as follows:

$$\delta p = \begin{cases} \delta n + \delta n_T, & \text{for the system with the electron trap,} \\ \delta n - \delta p_T, & \text{for the system with the hole trap.} \end{cases} \quad (16)$$

In a system dominated by the transition of the total free carriers through the localized center, the rate of accumulation of free electrons in the conduction band during illumination is given by

$$\frac{dn}{dt} = G + U_{\text{en}} - U_{\text{cn}}. \quad (17)$$

The total generation rate  $G$  in equation (17) involves both the thermal generation rate  $G_{\text{th}}$  and the illumination rate  $G_0$  (or  $G = G_{\text{th}} + G_0$ ), and the rates of capture  $U_{\text{cn}}$  and emission  $U_{\text{en}}$  of both thermal and excess electrons by the localized state are given by

$$U_{\text{cn}} = U_{\text{cn}}^0 + C_{\text{nT}} [p_{0T} \delta n - (n_0 + \delta n) \delta n_T], \quad (18)$$

$$U_{\text{en}} = U_{\text{en}}^0 + C_{\text{nT}} n_1 \delta n_T. \quad (19)$$

These two relations are similar to the one derived by Macdonald and Cuevas and Wertheim [15, 24]. Applying the principle of detailed balance to equation (17) gives the rate of accumulation of excess electrons at the localized state to be

$$\frac{d\delta n}{dt} = \begin{cases} G_0 - \frac{p_{0T} \delta n - (n_0 + n_1) \delta n_T}{\tau_{n0} N_T}, \\ G_0 - \left( U_{\text{SRH}} + \frac{d\delta n_T}{dt} \right), \end{cases} \quad (20)$$

and the rate of accumulation of excess holes in the valence band is determined following a similar procedure and

applying the neutrality relations in equations (15) and (16) as follows:

$$\frac{d\delta p}{dt} = \begin{cases} G_0 - \frac{n_{0T} \delta n + (p_0 + p_1 + n_{0T}) \delta n_T}{\tau_{p0} N_T}, \\ G_0 - U_{\text{SRH}}, \end{cases} \quad (21)$$

where  $\tau_{n0}$  and  $\tau_{p0}$  are the values of the excess carrier lifetimes at the recombination center when the localized state is empty ( $p_{0T} = N_T$ ) and filled completely ( $n_{0T} = N_T$ ), respectively, as we shall see later, given by [1]

$$\begin{aligned} \tau_{n0} &= \frac{1}{C_{\text{nT}} N_T}, \\ \tau_{p0} &= \frac{1}{C_{\text{pT}} N_T}. \end{aligned} \quad (22)$$

Applying the neutrality relations in equations (15) and (16) to relation (21) give the value of the SRH recombination rate  $U_{\text{SRH}}$  at the localized state to be

$$U_{\text{SRH}} = \begin{cases} \frac{(p_0 + p_1 + n_{0T}) \delta p - (p_0 + p_1) \delta n}{\tau_{p0} N_T}, \\ \frac{\delta n}{\tau_{\text{nSRH}}} = \frac{\delta p}{\tau_{\text{pSRH}}}, \end{cases} \quad (23)$$

where  $\tau_{\text{nSRH}}$  and  $\tau_{\text{pSRH}}$  are the SRH recombination excess electron and hole lifetimes. One can see that, for a system with the electron trap, the last term on the right side of equation (20) contains the SRH recombination rate  $U_{\text{SRH}}$  of the excess carriers through the localized states and the accumulation rates of excess electrons at the localized state, while the last term on the right side of equation (21) contains only  $U_{\text{SRH}}$ .

The rate of accumulation of free electrons at the localized center is obtained by subtracting equation (20) from equation (21) as

$$\frac{d\delta n_T}{dt} = U_{T\delta n} - U_{E\delta n}, \quad (24)$$

where

$$U_{T\delta n} = \frac{\delta n}{\tau_{\text{nt}}}, \quad (25)$$

$$U_{E\delta n} = \frac{\delta n_T}{\tau_{\text{ne}}},$$

are the trapping and the emission rates of excess electrons by the localized state, respectively, and  $\tau_{\text{nt}}$  and  $\tau_{\text{ne}}$  are the respective lifetimes corresponding to the trapping and emission of excess electrons by the localized state given by

$$\tau_{\text{nt}} = \frac{N_T \tau_{n0} \tau_{p0}}{\tau_{p0} p_{0T} - \tau_{n0} n_{0T}}, \quad (26)$$

$$\tau_{\text{ne}} = \frac{N_T \tau_{n0} \tau_{p0}}{\tau_{p0} (n_0 + n_1) + \tau_{n0} (p_0 + p_1 + n_{0T})}. \quad (27)$$

The rate of accumulation of excess holes at the localized state in a system with the hole trap is also described using a similar procedure:

$$\frac{d\delta p_T}{dt} = U_{T\delta p} - U_{E\delta p}, \quad (28)$$

where the respective trapping and the emission rates of excess holes by the localized state are described as

$$U_{T\delta p} = \frac{\delta p}{\tau_{pt}}, \quad (29)$$

$$U_{E\delta p} = \frac{\delta p_T}{\tau_{pe}},$$

and  $\tau_{pe}$  and  $\tau_{pt}$  are lifetimes corresponding to the emission and the trapping of excess holes by the localized state, respectively, given by

$$\tau_{pt} = \frac{N_T \tau_{n0} \tau_{p0}}{\tau_{n0} n_{0T} - \tau_{p0} p_{0T}}, \quad (30)$$

$$\tau_{pe} = \frac{N_T \tau_{n0} \tau_{p0}}{\tau_{p0} (n_0 + n_1 + p_{0T}) + \tau_{n0} (p_0 + p_1)}. \quad (31)$$

The steady-state conditions of equations (21), (24), and (28) give

$$\begin{aligned} U_{SRH} &= G_0 \\ &= \frac{\delta n}{\tau_{nSRH}} \\ &= \frac{\delta p}{\tau_{pSRH}}, \end{aligned} \quad (32)$$

$$\begin{aligned} U_{T\delta n} &= U_{E\delta n} \\ &= \frac{\delta n}{\tau_{nt}} \\ &= \frac{\delta n_T}{\tau_{ne}}, \end{aligned} \quad (33)$$

$$\begin{aligned} U_{T\delta p} &= U_{E\delta p} \\ &= \frac{\delta p}{\tau_{pt}} \\ &= \frac{\delta p_T}{\tau_{pe}}. \end{aligned} \quad (34)$$

Using these steady-state conditions in equations (32)–(34) along with the steady state conditions of the upper parts of equations (20) and (21), we get

$$\delta n = \frac{\tau_{n0} (p_0 + p_1 + n_{0T}) + \tau_{p0} (n_0 + n_1)}{(p_0 + n_0) N_T + p_{0T} n_{0T}} N_T G_0, \quad (35)$$

$$\delta n_T = \frac{\tau_{p0} p_{0T} - \tau_{n0} n_{0T}}{(p_0 + n_0) N_T + p_{0T} n_{0T}} N_T G_0. \quad (36)$$

Addition of equations (35) and (36) gives the expression for the excess hole density in the valence band:

$$\delta p = \frac{\tau_{n0} (p_0 + p_1) + \tau_{p0} (n_0 + n_1 + p_{0T})}{(p_0 + n_0) N_T + p_{0T} n_{0T}} N_T G_0. \quad (37)$$

Dividing  $\delta n$  and  $\delta p$  in equations (35) and (37) by  $G_0$  hence yields

$$\tau_{nSRH} = \frac{\tau_{n0} (p_0 + p_1 + n_{0T}) + \tau_{p0} (n_0 + n_1)}{(p_0 + n_0) N_T + p_{0T} n_{0T}} N_T, \quad (38)$$

$$\tau_{pSRH} = \frac{\tau_{n0} (p_0 + p_1) + \tau_{p0} (n_0 + n_1 + p_{0T})}{(p_0 + n_0) N_T + p_{0T} n_{0T}} N_T. \quad (39)$$

The steady-state values of the excess electron and hole trapping rates  $U_{T\delta n}$  and  $U_{T\delta p}$  in equations (33) and (34) are hence described in terms of  $G_0$  as

$$U_{T\delta n} = \left\{ 1 + \frac{(\tau_{p0}^2 n_1 - 2\tau_{n0} \tau_{p0} n_0 - \tau_{n0}^2 p_1) N_T - \tau_{n0}^2 n_{0T}^2}{\tau_{n0} \tau_{p0} [(p_0 + n_0) N_T + p_{0T} n_{0T}]} \right\} G_0, \quad (40)$$

$$U_{T\delta p} = \left\{ 1 + \frac{(\tau_{n0}^2 p_1 - 2\tau_{p0} \tau_{n0} p_0 - \tau_{p0}^2 n_1) N_T - \tau_{p0}^2 p_{0T}^2}{\tau_{p0} \tau_{n0} [(p_0 + n_0) N_T + p_{0T} n_{0T}]} \right\} G_0. \quad (41)$$

The ratios of the emission lifetimes to the trapping lifetimes corresponding to the excess electrons and holes at the localized states are given from equations (33) and (34) by

$$\frac{\tau_{ne}}{\tau_{nt}} = \frac{\delta n_T}{\delta n} = \gamma_n, \quad (42)$$

$$\frac{\tau_{pe}}{\tau_{pt}} = \frac{\delta p_T}{\delta p} = \gamma_p,$$

where  $\gamma_n$  and  $\gamma_p$  are very important quantities named as the excess electron and hole trapping ratios described as

$$\gamma_n = \frac{\tau_{p0} p_{0T} - \tau_{n0} n_{0T}}{\tau_{n0} (p_0 + p_1 + n_{0T}) + \tau_{p0} (n_0 + n_1)}, \quad (43)$$

$$\gamma_p = \frac{\tau_{n0} n_{0T} - \tau_{p0} p_{0T}}{\tau_{n0} (p_0 + p_1) + \tau_{p0} (n_0 + n_1 + p_{0T})}. \quad (44)$$

When the definitions of free-carrier trapping ratios in equation (42) are applied to the neutrality relations in equation (16) for a system with the electron trapping center, we have

$$\delta p = (1 + \gamma_n) \delta n = \frac{1 + \gamma_n}{\gamma_n} \delta n_T. \quad (45)$$

Following the same procedure for a system with the hole trapping center, we get

$$\delta n = (1 + \gamma_p) \delta p = \frac{1 + \gamma_p}{\gamma_p} \delta p_T. \quad (46)$$

Dividing the steady-state value of  $\delta p$  and  $\delta n$  in equations (45) and (46) by  $G_0$  gives the relation between SRH lifetimes

for holes and electrons in terms of the trapping ratios to be as follows (see equation (32)):

$$\tau_{pSRH} = \begin{cases} (1 + \gamma_n)\tau_{nSRH}, & \text{for the system with the electron trap,} \\ \frac{\tau_{nSRH}}{1 + \gamma_p}, & \text{for the system with the hole trap.} \end{cases} \quad (47)$$

Taking into account equations (33), (34), and (42) along with the neutrality relation in equation (15) also yields

$$\tau_{pe} = \begin{cases} \frac{\tau_{ne}}{1 + \gamma_n}, & \text{for the system with the electron trap,} \\ \tau_{ne}(1 + \gamma_p), & \text{for the system with the hole trap.} \end{cases} \quad (48)$$

### 3. Model and Methods

Since the low injection level excess carrier lifetimes ( $\tau_{nt}$ ,  $\tau_{ne}$ ,  $\tau_{pt}$ ,  $\tau_{pe}$ ,  $\tau_{nSRH}$ , and  $\tau_{pSRH}$ ) in equations (26), (27), (30), (31), (38), and (39) and the free-carrier trapping ratios ( $\gamma_n$  and  $\gamma_p$ ) in equations (43) and (44) are independent of the illumination rate,  $G_0$ , they describe the thermal equilibrium properties of the material. The equations governing these quantities are varying as functions of the doping level, density of the localized states, and the localized state energies between the valence band maximum,  $E_V$ , and the conduction band minimum,  $E_C$ . That is, except  $p_0$ ,  $n_0$ ,  $n_i$ , and  $N_T$ , all the quantities (variables) involved in all the expressions above are varying with the energies of the localized states. In order to study the variations of these quantities with the localized state energies in the bandgap of the material, we kept the values of the densities of all the localized states  $N_T$  to be constant at  $10^{17} \text{ cm}^{-3}$  and the density of the shallow acceptor  $N_a$  to be  $10^{18} \text{ cm}^{-3}$ . To avoid the effects of carriers' compensation, the magnitudes of the deep-level localized state densities are made to be less than those of the acceptor density ( $N_T < N_a$ ) in the p-type semiconductor. The interaction among different localized states is also neglected, and only the interaction between each localized state and the two bands (valence and conduction) is considered. The equation governing the ionization density  $N_a^-$  of a shallow acceptor of density  $N_a$  and ionization energy  $\Delta E_a$  is adopted from our previous work [25]:

$$N_a^- = \frac{2N_a}{1 + \sqrt{1 + (4g_a N_a / N_V) \exp(\Delta E_a / k_B T)}}, \quad (49)$$

where  $g_a$  is the acceptor degeneracy and  $N_V$  is the effective density of the valence band states given by

$$N_V = 2 \left( \frac{2\pi m_p^* k_B T}{h^2} \right)^{3/2}, \quad (50)$$

and  $h$  is Planck's constant. The room temperature values of the ratios of  $m_p^*$  and  $m_n^*$  to the rest mass of the electron  $m_n$  for silicon adopted from other works are 1.115 and 1.08,

respectively [5]. The result of  $N_a^-$  in this work is determined at room temperature for a very shallow acceptor ( $\Delta E_a \approx 0$ ), and  $N_a = 10^{18} \text{ cm}^{-3}$ . The majority carrier density in the valence band  $p_0$  is also described in terms of the intrinsic carrier density  $n_i$  and the acceptor ionization density  $N_a^-$  by applying the well-known neutrality relation of one acceptor level system.

$$p_0 = \frac{N_a^-}{2} + \sqrt{n_i^2 + \left(\frac{N_a^-}{2}\right)^2}, \quad (51)$$

and the minority carrier density is also obtained by applying the law of mass action,  $n_0 = n_i^2 / p_0$ .

As one can see from equations (45)–(48), the presence of the carrier trapping ratios  $\gamma_n$  or  $\gamma_p$  resulted in the variations of the values of the excess carrier densities and the excess carrier lifetimes related to the majority and minority carrier trapping and recombination rates. Hence, detailed description of the steady-state values of  $\gamma_n$  or  $\gamma_p$  at different localized states in the bandgap of the semiconductor also helps us to describe the role of localized states in controlling the performance of the material. One can also see that the normalized forms of the free-carrier transition rates  $U_{SRH}/G_0$ ,  $U_{T\delta n}/G_0$ , and  $U_{T\delta p}/G_0$  in equations (32), (40), and (41) are also independent of the illumination. The localized states that are playing the dominance role in the transfer of free carriers can be identified upon describing these normalized carrier transition rates as functions of localized state energies.

### 4. Results and Discussion

**4.1. Trapping Ratios.** The localized state, in general, can be the excess electron trap center only when  $\gamma_n > 0$  (or  $\tau_{p0} p_{0T} > \tau_{n0} n_{0T}$ ) and excess hole trap center when  $\gamma_p > 0$  (or  $\tau_{p0} p_{0T} < \tau_{n0} n_{0T}$ ). Deep localized state with  $\gamma_n = \gamma_p \rightarrow 0$  is said to be a recombination center, and a shallow localized state with  $\gamma_n = \gamma_p \rightarrow 0$  is said to be an acceptor or donor level. Figure 2 depicts the room-temperature localized state energy dependence of the semilog calibration of excess electron and hole trapping ratios  $\gamma_p$  (red curve) and  $\gamma_n$  (blue curve) in p-type silicon (Si) with a very shallow acceptor ( $\Delta E_a \approx 0$ ) and  $N_a = 10^{18} \text{ cm}^{-3}$  and deep localized states of uniform densities  $N_T = 10^{17} \text{ cm}^{-3}$  in the entire bandgap of the material. The blue and the red curves in Figure 2 represent the values of  $\gamma_n$  and  $\gamma_p$ , respectively. The horizontal line (with wine color) is drawn to show where the trapping ratios attain the values equal to unity. The semilog graphs of  $\gamma_n$  are drawn using equation (43) for the case when  $\tau_{p0} p_{0T} > \tau_{n0} n_{0T}$  and those of  $\gamma_p$  are drawn using equation (44) for the case when  $\tau_{p0} p_{0T} < \tau_{n0} n_{0T}$ . Depending on the interactions of the localized states with the conduction or valence bands, the region under the localized state energy dependence of the excess carrier trapping ratio in Figure 2 can be divided into four regions labelled I through IV.

**4.1.1. Region I (Acceptor Region).** This region comprises all the shallow localized states of activation energies less than 100 meV relative to the valence band edge,  $E_V$ . Most of the

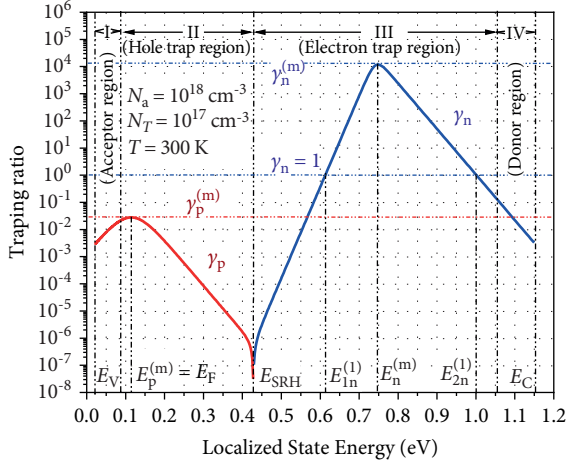


FIGURE 2: Room-temperature localized state energy dependence of  $\gamma_p$  (red curve) and  $\gamma_n$  (blue curve) for p-type Si with shallow acceptor density  $1018 \text{ cm}^{-3}$  and localized state density  $1017 \text{ cm}^{-3}$ .

impurities are in the  $n_T$  state (i.e.,  $n_{0T} \approx N_T$ ), and the hole emission rate is very high (i.e.,  $p_1 \gg p_0, N_T, n_1$ ). The hole capture coefficient is very much greater than the electron capture coefficient ( $C_{pT} \gg C_{nT}$ ,  $\tau_{n0}n_{0T} \gg \tau_{p0}p_{0T}$ , or  $\gamma_n = 0$ ). The hole trapping ratio  $\gamma_p$  in equation (44) in this region is also negligibly very small and simplified to

$$\gamma_p \approx \frac{N_T}{p_1} \approx 3 \times 10^{-3}. \quad (52)$$

When equation (52) is taken into account along with equation (46) for a system with the hole trap center, we have  $\delta p = 0.997\delta n$ , and hence,  $\delta p_T = 0.003\delta n$ . This shows that only about 0.3% of the density of excess holes is trapped at this localized state as compared to the density of the generated electron-hole pairs, and 99.7% of the excess holes are in the valence band. The region hence acts as an acceptor region.

**4.1.2. Region II (Hole Trap Region).** Using the approximation  $\tau_{n0}n_{0T} \gg \tau_{p0}p_{0T}$  in the region far below the midgap, the value of the hole trapping ratio  $\gamma_p$  in equation (44) is simplified to

$$\gamma_p \approx \frac{n_{0T}}{p_0 + p_1}. \quad (53)$$

Upon using the values of  $n_{0T}$  and  $p_1$  in equations (2) and (11) and optimizing equation (53) as a function of energy,  $E_T$ , the energy  $E_p^{(m)}$  at which  $\gamma_p$  attains its maximum is obtained to be

$$E_p^{(m)} \approx E_F, \quad (54)$$

and the maximum value of  $\gamma_p$  at this energy is also found to be

$$\gamma_p^{(m)} \approx \frac{N_T}{4p_0} < 1. \quad (55)$$

The energy  $E_p^{(m)}$  at which  $\gamma_p$  attains its maximum value  $\gamma_p^{(m)}$  is situated below the midgap very close to the Fermi energy level ( $p_1 \approx p_0 \gg n_i$ ). Hence, one can see that most localized states in this region are partially filled (i.e.,  $n_{0T} \approx p_{0T} = N_T/2$ ). The value of  $\gamma_p^{(m)}$  in equation (55) in this case is about  $2.8 \times 10^{-2}$  which is also very small. Using this along with equation (46) for a system with the hole trapping center, we have  $\delta p = 0.973\delta n$  and  $\delta p_T = 0.027\delta n$ . This means, among the total density of the generated electron-hole pairs, about 2.7% of excess hole densities are trapped at this localized state, and about 97.3% of the free hole densities remain in the valence band.

With increasing the energy of the localized states,  $E_T$ , the value of  $\gamma_p$  decreases from its maximum value  $\gamma_p^{(m)}$  in equation (55) to zero at the localized state of energy:

$$E_{\text{SRH}} = \frac{E_V + E_C}{2} - \frac{k_B T}{12} \ln \left[ \frac{m_n^*}{m_p^*} \left( \frac{p_0}{n_i} \right)^4 \right], \quad (56)$$

as shown in Figure 2. Since  $\tau_{p0}p_{0T} \approx \tau_{n0}n_{0T}$  at this localized state, the values of both  $\gamma_n$  and  $\gamma_p$  in equations (43) and (44) are zero, and thus, no excess carriers are trapped at this localized state,  $\delta n_T = -\delta p_T = 0$  (see equation (36)). Therefore, all the captured excess carriers by this localized state are recombined with the opposite excess carriers. Hence, the localized state can be considered as the SRH recombination center and represented by  $E_{\text{SRH}}$ . Shallow localized states with energies between the acceptor region and the SRH recombination center can hence be considered as the hole trapping region. One can see from equation (56) that  $E_{\text{SRH}}$  is always located below the midgap in highly doped p-type material ( $p_0 \gg n_i$ ) and moves farther away from the midgap with increasing the majority carrier density.  $E_{\text{SRH}}$  can coincide with the midgap only if  $p_0 = n_i \sqrt[4]{m_p^*/m_n^*}$ . The location of  $E_{\text{SRH}}$  in this case is approximately at 0.405 eV above the valence band maximum or 0.157 eV below the midgap.

**4.1.3. Region III (Electron Trap Region).** Electron trapping region covers the regions above the SRH recombination center in which most localized states are in the  $p_T$  state (i.e.,  $p_{0T} \approx N_T$ ), and the electron trap effect becomes the most dominant for  $\tau_{p0}p_{0T} \gg \tau_{n0}n_{0T}$ . Using these facts along the above midgap approximations ( $p_0 \gg p_1$  and  $n_1 \gg n_0, n_{0T}$ ), one can simplify the expression for the electron trapping ratio  $\gamma_n$  in equation (43) in this region as

$$\gamma_n \approx \frac{\tau_{p0}N_T}{\tau_{n0}p_0 + \tau_{p0}n_1}, \quad (57)$$

and with increasing the localized state energy  $E_T$  above the SRH recombination center, the electron trapping ratio  $\gamma_n$  increases from its zero value at energy  $E_{\text{SRH}}$  until it attains its value equal to unity (1) at the localized state of energy:

$$E_{1n}^{(1)} = \frac{E_V + E_C}{2} + \frac{k_B T}{4} \ln \left[ \frac{m_n^*}{m_p^*} \left( \frac{p_0}{N_T} \right)^2 \right]. \quad (58)$$

One can see from equation (58) that  $E_{1n}^{(1)}$  at which  $\gamma_n$  attains its first value 1 above the energy  $E_{\text{SRH}}$  is located above the midgap in highly doped p-type material ( $p_0 \gg N_T$ ). When equation (45) is taken into account, we have  $\delta n = 0.5\delta p$ , and hence,  $\delta n_T = 0.5\delta p$ . This shows that, among the generated excess electron-hole pairs, only 50% is in the conduction band, and 50% is trapped at this localized state. With further increase in the localized state energy  $E_T$  beyond  $E_{1n}^{(1)}$ , the electron trapping ratio  $\gamma_n$  increases and reaches its maximum value:

$$\gamma_n^{(m)} \approx \frac{N_T}{3p_0} \left( \frac{2p_0}{n_i} \frac{m_p^*}{m_n^*} \right)^{2/3} \quad (59)$$

at the localized state of energy:

$$E_n^{(m)} \approx \frac{E_V + E_C}{2} + \frac{k_B T}{12} \ln \left[ \frac{m_p^*}{m_n^*} \left( \frac{2p_0}{n_i} \right)^4 \right]. \quad (60)$$

The pick values  $\gamma_n^{(m)}$  and  $E_n^{(m)}$  in equations (59) and (60) are obtained by optimizing equation (57) in terms of energy  $E_T$  (after describing  $\tau_{n0}$ ,  $\tau_{p0}$ , and  $n_1$  in terms of  $E_T$ ). Upon applying the properties of p-type materials ( $m_p^* > m_n^*$  and  $p_0 \gg n_i$ ) to equations (59) and (60), one can see that  $\gamma_n^{(m)}$  is extremely larger than unity, and the energy  $E_n^{(m)}$  is always situated far above the midgap energy. The location of  $E_n^{(m)}$  is approximately at 0.163 eV above the midgap and 0.400 eV below the conduction band edge in this case. Relation (59) also gives a very large value of  $\gamma_n^{(m)}$  which is about  $1.19 \times 10^4$  in this case. When equation (45) is taken into account in this case, we have  $\delta n = 0.0001\delta p$ , and hence,  $\delta n_T = 0.9999\delta p$ . This shows that, among the total density of excess electron-hole pairs generated, only about 0.01% of the density of excess electrons remained in the conduction band, and about 99.99% of the density of excess electrons are trapped at this localized state. Hence, this localized center and the nearby regions are acting as electron trapping regions. As the localized state energy  $E_T$  is increased above  $E_n^{(m)}$ , the value of  $\gamma_n$  decreases from its maximum value described in equation (59) to unity (1) at the energy

$$E_{2n}^{(1)} = E_i + k_B T \ln \left( \frac{N_T}{n_i} \right), \quad (61)$$

as shown on the right side of Figure 2. The energy  $E_{2n}^{(1)}$  in equation (61) is situated above the intrinsic energy level  $E_i$  when  $N_T > n_i$ . In this case, it is located approximately at 0.415 eV above the intrinsic energy level  $E_i$  and at 0.146 eV below the conduction band edge  $E_C$ . As the localized state energy  $E_T$  is increased beyond  $E_{2n}^{(1)}$  the value of  $\gamma_n$  diminishes to very small fraction.

**4.1.4. Region IV (Donor Region).** This region also comprises all the shallow localized states of activation energies less than 100 meV relative to the conduction band edge  $E_C$ . Most of the impurities are in the  $p_T$  state (i.e.,  $p_{0T} \approx N_T$ ). The hole capture rate in this region is negligibly small

( $\tau_{p0} p_{0T} \gg \tau_{n0} n_{0T}$ ), and the electron emission rate is extremely very high ( $n_1 \gg p_0, N_T$ ). Hence, the excess electron trapping ratio  $\gamma_n$  in equation (43) is simplified at the conduction band edge  $E_C$  to

$$\gamma_n \approx \frac{N_T}{n_1} = 3.55 \times 10^{-3}. \quad (62)$$

Again, when equation (45) is taken into account along with equation (62), we get  $\delta n = 0.9965\delta p$ , and hence,  $\delta n_T = 0.0035\delta p$ . This implies that 99.65% of the density of the generated excess electrons are in the conduction band, and only about 0.35% of the density of the excess electrons are trapped in this region. The region is hence called the donor region.

**4.2. Excess Carrier Lifetimes.** Next, the variations of the excess carrier lifetimes (trapping, emission, and recombination lifetimes) with the localized state energies in the bandgap of the semiconductor are described in detail. The variations of the excess carrier lifetimes with the localized state energies are drawn using equations (26), (27), (30), (31), (38), and (39). Figure 3 depicts the room-temperature localized state energy dependence of the semilog displays of the excess carrier lifetimes corresponding to (a) the emission of excess electrons and holes by the localized states,  $\tau_{ne}$  and  $\tau_{pe}$ , (b) the trapping of excess electrons and holes by the localized states,  $\tau_{nt}$  and  $\tau_{pt}$ , and (c) the SRH lifetimes for electrons and holes,  $\tau_{n\text{SRH}}$  and  $\tau_{p\text{SRH}}$ , at the localized states in p-type Si discussed in Figure 2. The blue and the red curves in Figure 3 represent the lifetimes corresponding to the excess electrons and holes, respectively. Since  $\tau_{p0}$  and  $\tau_{n0}$  are varying oppositely with the variation of the localized state energies, they are drawn for the reference in Figures 3(a)–3(c). The plots under the curves in Figures 3(a) and 3(c) can be divided into five distinct regions labelled I, II, III, IV, and V, while the plot under Figure 3(c) can be divided into three regions labelled I, II, and III. As one can see from Figures (a) and (b), the values of  $\tau_{ne}$ ,  $\tau_{pe}$ ,  $\tau_{pt}$ , and  $\tau_{nt}$  increase from the shallow localized state regions (regions I and V) towards the deep localized states (regions III and IV) contrarily to the values of  $\tau_{n\text{SRH}}$  and  $\tau_{p\text{SRH}}$  depicted in Figure 3(c).

Near the valence band edge  $E_V$ , when the near valence-band approximations ( $\gamma_p \rightarrow 0$ ,  $n_{0T} \approx N_T$  and  $p_1 \gg p_0, N_T, n_1$ ) are applied to the expressions of the excess carrier lifetimes described in equations (27), (30), (31), (38), and (39), we obtained

$$\begin{aligned} \tau_{pt} &= \tau_{p0}, \\ \tau_{ne} &= \tau_{pe} \end{aligned} \quad (63)$$

$$\begin{aligned} &= \frac{N_T}{p_1} \tau_{p0}, \\ \tau_{n\text{SRH}} &= \tau_{p\text{SRH}} = \frac{p_1}{p_0} \tau_{n0}. \end{aligned} \quad (64)$$



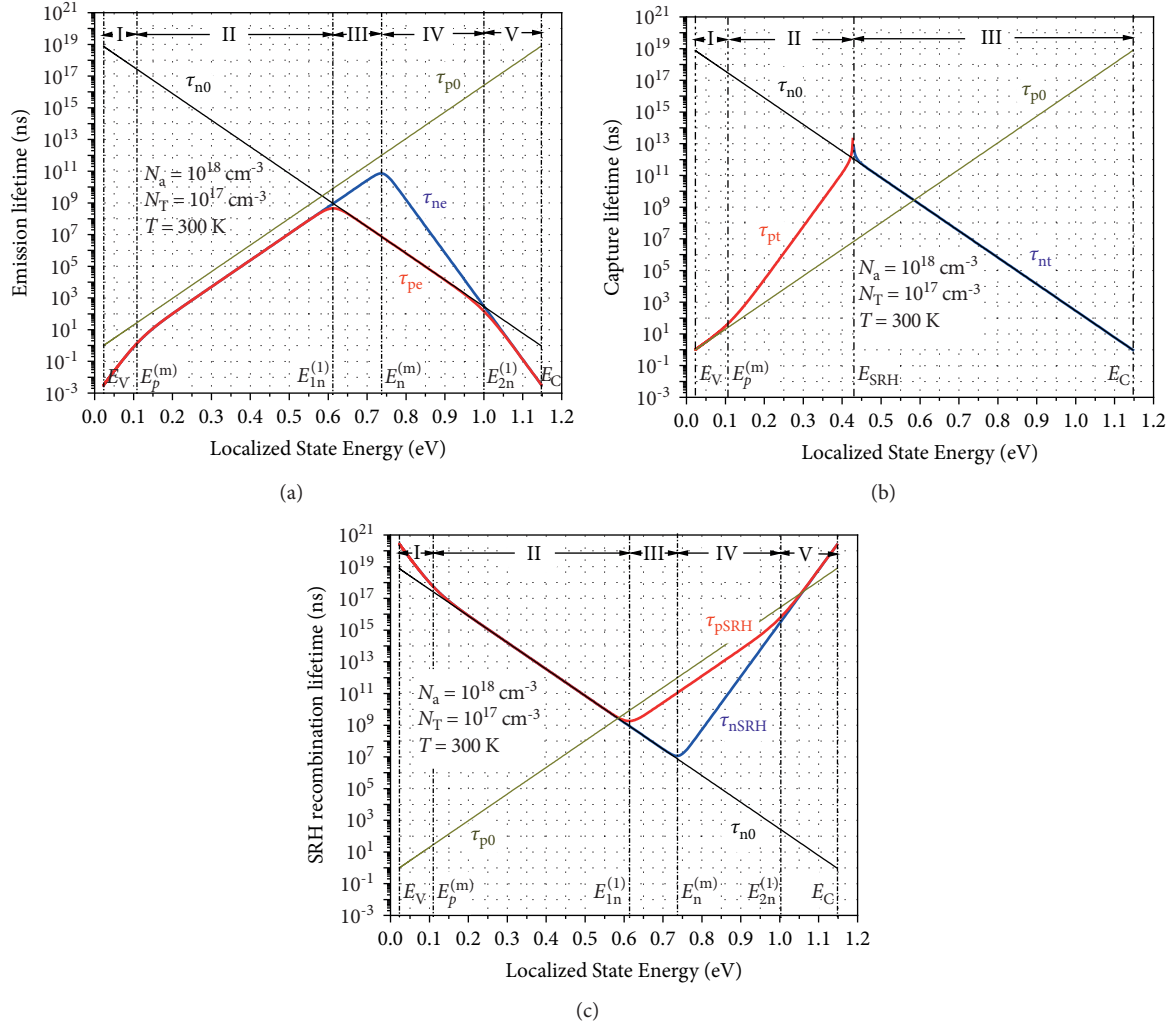


FIGURE 3: Room-temperature localized state energy dependence of (a)  $\tau_{ne}$  and  $\tau_{pe}$ , (b)  $\tau_{nt}$  and  $\tau_{pt}$ , and (c)  $\tau_{nSRH}$  and  $\tau_{pSRH}$  for the system discussed in Figure 2.

Since  $p_1 > N_T$  in this case,  $\tau_{pt} > \tau_{pe}$  as shown on the right sides of Figures 3(a) and 3(b).

Near the Fermi energy level,  $E_F$  (or  $E_p^{(m)}$ ), where  $p_1 = p_0$  and  $n_{0T} = p_{0T} = N_T/2$ , one can see that  $\gamma_p \ll 1$ . The expressions for the lifetimes in equations (27), (30), (31), (38), and (39) are hence simplified to

$$\begin{aligned} \tau_{pt} &= 2\tau_{p0}, \\ \tau_{ne} &= \tau_{pe} \end{aligned} \quad (65)$$

$$= \frac{N_T}{2p_0} \tau_{p0},$$

$$\tau_{nSRH} = \tau_{pSRH} = 2\tau_{n0}. \quad (66)$$

Since  $2P_0 > N_T$  in this case, still we see that  $\tau_{pt} > \tau_{pe}$  as depicted on the right sides of Figures 3(a) and 3(b).

Near the recombination center,  $E_{SRH}$ , we can apply the near midgap approximations  $p_{0T} \approx N_T$ ,  $p_0 \gg p_1$ ,  $n_1$  so that

$$\tau_{nt} = \tau_{pt} \longrightarrow \infty, \quad (67)$$

$$\tau_{ne} = \tau_{pe} \approx \frac{N_T}{p_0} \tau_{p0}.$$

$$\tau_{nSRH} = \tau_{pSRH} \approx \tau_{n0}. \quad (68)$$

One can see that  $\tau_{pt} \gg \tau_{pe}$  and  $\tau_{nt} \gg \tau_{ne}$  at  $E_{SRH}$  as shown in Figures 3(a) and 3(b).

Near the energy,  $E_{1n}^{(1)}$ , the value of the electron trapping ratio is equal to unity ( $\gamma_n = 1$ ). The expressions for the lifetimes in equations (26), (27), (31), (38), and (39) become

$$\tau_{ne} = \tau_{nt} = \tau_{nSRH} = 2\tau_{pe} = \frac{\tau_{pSRH}}{2} = \tau_{n0}. \quad (69)$$

Thus,  $\tau_{pe}$  attains a maximum value of  $\tau_{n0}/2$ , and  $\tau_{pSRH}$  attains a minimum value of  $2\tau_{n0}$  at energy  $E_{1n}^{(1)}$  as shown in Figures 3(a) and 3(c), respectively.

Near the energy,  $E_n^{(m)}$ , the excess electron trapping ratio  $\gamma_n$  reaches its maximum value given in equation (59). Applying this to equations (26), (27), (31), (38), and (39), we get

$$\begin{aligned}\tau_{pe} &= \tau_{nt} \\ &= \frac{\tau_{ne}}{1 + \gamma_n^{(m)}} \\ &= \tau_{n0}, \\ \tau_{pSRH} &= \tau_{nSRH}(1 + \gamma_n^{(m)}) \\ &= \frac{N_T}{p_0} \tau_{p0}.\end{aligned}\quad (70)$$

The lifetime related to the emission of electrons by the localized state  $\tau_{ne}$  attains the maximum value of  $(1 + \gamma_n^{(m)})\tau_{n0}$  and that related to the electron SRH recombination  $\tau_{nSRH}$  attains the minimum value of  $(1 + \gamma_n^{(m)})^{-1}N_T\tau_{p0}/p_0$  at energy  $E_n^{(m)}$  as depicted in Figures 3(a) and 3(c).

Near the energy,  $E_{2n}^{(1)}$ , when the value of  $\gamma_n = 1$  is used along with the approximations  $p_{0T} = N_T$  and  $\tau_{p0}p_{0T} \gg \tau_{n0}n_{0T}$  in equations (26), (27), (31), (38), and (39), we get the values

$$\begin{aligned}\tau_{ne} &= \tau_{nt} \\ &= 2\tau_{pe} \\ &= \tau_{n0}, \\ \tau_{pSRH} &= 2\tau_{nSRH} \\ &= \frac{2n_1}{p_0} \tau_{p0}.\end{aligned}\quad (72)$$

Here, we see that  $\tau_{ne} > \tau_{pe}$  and  $\tau_{pSRH} > \tau_{nSRH}$  as illustrated at  $E_{2n}^{(1)}$  on the right side of Figures 3(a) and 3(c).

Near the conduction band,  $E_C$  (in the donor region), we apply the approximations  $p_{0T} = N_T$ ,  $n_1 > p_0$ , and  $\tau_{p0} \gg \tau_{n0}$ , and thus,

$$\begin{aligned}\tau_{nt} &= \tau_{n0}, \\ \tau_{ne} &= \tau_{pe} \\ &= \frac{N_T}{n_1} \tau_{n0}, \\ \tau_{nSRH} &= \tau_{pSRH} \\ &= \frac{n_1}{p_0} \tau_{p0}.\end{aligned}\quad (74)$$

Since  $N_T < n_1$  in the equation in this case,  $\tau_{nt} \gg \tau_{ne}$  as shown on the right sides of Figures 3(a) and 3(b).

Since the value of the hole trapping ratio is always less than unity ( $\gamma_p \ll 1$ ) as depicted in Figure 2, when equations (47) and (48) are taken into account in this case, we have  $\tau_{pe} \approx \tau_{ne}$  and  $\tau_{pSRH} \approx \tau_{nSRH}$ . Thus, the majority carrier trap

cannot affect the values of the emission lifetimes ( $\tau_{pe}$  and  $\tau_{ne}$ ) and the SRH recombination lifetimes ( $\tau_{pSRH}$  and  $\tau_{nSRH}$ ) of the majority and minority carriers as shown in Figures 3(a) and 3(c). In the localized states of energies below  $E_{1n}^{(1)}$  and above  $E_{2n}^{(1)}$ , the value of the electron trapping ratio is less than unity ( $\gamma_n \ll 1$ ) as depicted in Figure 2, and thus, when equations (47) and (48) are considered in this case, we see that  $\tau_{pe} \approx \tau_{ne}$  and  $\tau_{pSRH} \approx \tau_{nSRH}$  as shown in Figures 3(a) and 3(c). This also shows that the minority carrier trapping cannot affect the free-carrier lifetimes in the regions below  $E_{1n}^{(1)}$  and above  $E_{2n}^{(1)}$ . Between the localized states of energies  $E_{1n}^{(1)}$  and  $E_{2n}^{(1)}$  where  $\gamma_n > 1$ , the SRH lifetime of free electrons  $\tau_{nSRH}$  shown in Figure 3(c) is decreased by a factor of  $1 + \gamma_n$  as compared to the SRH lifetime of free holes  $\tau_{pSRH}$ . Nevertheless, the free-electron emission lifetime  $\tau_{ne}$  by the localized states between  $E_{1n}^{(1)}$  and  $E_{2n}^{(1)}$  is increased by a factor of  $1 + \gamma_n$  as compared to the emission lifetime of free holes by the localized states  $\tau_{pe}$  as shown in Figure 3(a). This shows that minority carrier traps in the localized states between  $E_{1n}^{(1)}$  and  $E_{2n}^{(1)}$  in the p-type semiconductor reduce the lifetime of free electrons in the conduction band and enhance the lifetime of holes in the valence band as reported by other researchers [1–5, 15]. Shallow and deep traps of energies below  $E_{1n}^{(1)}$  and above  $E_{2n}^{(1)}$  cannot affect the excess carrier lifetimes of free carriers in the p-type semiconductor.

**4.3. Dominant Transition Rates.** Since the excess carriers in the conduction and the valence bands are directly involved in the rates of the recombination and the trapping at the localized states, the SRH recombination and the trapping rates at the localized states are competing. The dominance of one transition rate of excess carriers over the other can be described directly by evaluating the variations of the SRH recombination and the trapping rates of excess carriers over the given ranges of energy. The dominant excess carrier transition rate at a given localized state can be identified using the graphs of the localized state energy dependence of the excess carrier trapping rates ( $U_{T\delta n}$  and  $U_{T\delta p}$ ) in equations (40) and (41) and the SRH recombination rate  $U_{SRH}$  in equation (32). The excess carrier transition rate with the largest value among  $U_{T\delta n}$ ,  $U_{T\delta p}$ , and  $U_{SRH}$  is considered as the dominant. Figure 4 shows the room-temperature localized state energy dependence of the semilog drawings of (a) the normalized trapping rates of excess electrons (blue curve) and holes (red curve) by the localized states and the normalized SRH recombination rate (green line) and (b) the excess carrier lifetimes corresponding to the trapping of excess electrons and holes by the localized states  $\tau_{nt}$  (blue curve) and  $\tau_{pt}$  (red curve) and the SRH lifetimes for electrons and holes  $\tau_{nSRH}$  (green curve) and  $\tau_{pSRH}$  (wine curve) through the localized states for the system discussed in Figure 2.

The energy level dependence of the curves of the excess carriers' transition rates in Figure 4(a) can also be divided into three main regions labelled I to III. Region I in Figure 4(a) is the combination of regions I and II in Figure 2. Near  $E_V$ , the normalized trapping rates of excess holes by the localized state described in equation (41) acquire the following extremely very large value (where  $p_1 \gg p_0$  and  $\tau_{n0} \gg \tau_{p0}$ ):

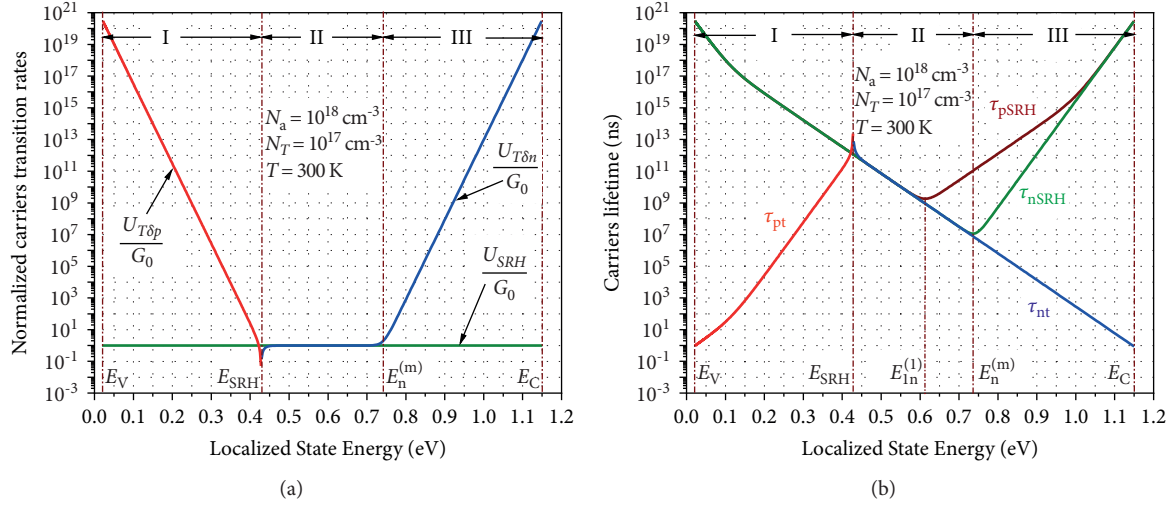


FIGURE 4: Room-temperature localized state energy dependence of (a)  $U_{T\delta n}/G_0$  (blue curve),  $U_{T\delta p}/G_0$  (red curve), and  $U_{\text{SRH}}/G_0$  (green curve) and (b)  $\tau_{\text{nt}}$  (blue curve),  $\tau_{\text{pt}}$  (red curve),  $\tau_{\text{nSRH}}$  (green curve), and  $\tau_{\text{pSRH}}$  (wine curve) for the system discussed in Figure 2.

$$\frac{U_{T\delta p}}{G_0} \approx \frac{\tau_{n0}}{\tau_{p0}} \frac{p_1}{p_0} \gg 1. \quad (76)$$

The values of the normalized hole trapping rate described above decrease with increasing the localized state energy (with decreasing the values of  $p_1/p_0$  and  $\tau_{n0}/\tau_{p0}$ ) in region I and become zero at the recombination center  $E_{\text{SRH}}$  as depicted in Figure 4(a). Hence, when the hole trapping rate by the localized states  $U_{T\delta p}/G_0$  is compared with that of normalized  $U_{\text{SRH}}$  (which is unity) in region I of Figure 4(a), the hole trapping rate by the localized states  $U_{T\delta p}$  is greater than the SRH recombination rate  $U_{\text{SRH}}$ . Thus,  $U_{T\delta p}$  dominates the entire excess carrier transition process in all the localized states in region I of Figure 4(a). The SRH recombination rate dominates the free-carrier transition rate at the energy  $E_{\text{SRH}}$ , where the values of  $U_{T\delta p}$  and  $U_{T\delta n}$  diminish to zero as shown in Figure 4(a).

Region II in Figure 4(a) covers the localized state energies between the recombination center  $E_{\text{SRH}}$  and the energy of the maximum excess electron trapping ratio  $E_n^{(m)}$ . The magnitude of the normalized trapping rates of excess electrons by the localized states  $U_{T\delta n}/G_0$  in this region approaches unity or  $U_{T\delta n} \approx G_0$ . Hence, both  $U_{T\delta n}$  and  $U_{\text{SRH}}$  are dominating the transition process of excess carriers equally in all the localized states in region II of Figure 4(a).

Region III in Figure 4(a) covers all the localized states between the energy  $E_n^{(m)}$  and the conduction band minimum  $E_C$ . The normalized excess electron trapping rate  $U_{T\delta n}/G_0$  described using equation (40) increases from its unity value near  $E_n^{(m)}$  to its extremely very large value near the conduction band minimum  $E_C$  (where  $n_1 \gg p_0$  and  $\tau_{p0} \gg \tau_{n0}$ ):

$$\frac{U_{T\delta n}}{G_0} = \frac{\tau_{p0}}{\tau_{n0}} \frac{n_1}{p_0} \gg 1. \quad (77)$$

Hence,  $U_{T\delta n}$  dominates the entire transition process of excess carriers through the localized states in region III as shown in Figure 4(a).

Since the free-carrier recombination and trapping rates at the localized states described in equations (32)–(34) involve the free-carrier densities in the conduction and valence bands, the dominant recombination rate can also be evaluated by comparing the reciprocal values of the excess carrier lifetimes  $\tau_{\text{nt}}$ ,  $\tau_{\text{pt}}$ ,  $\tau_{\text{nSRH}}$ , and  $\tau_{\text{pSRH}}$  described in equations (26), (30), (38), and (39), respectively. The excess carrier lifetime with the smallest value is hence considered to be the dominant excess carrier lifetime corresponding to the dominant transition rate. The region under the localized state energy dependence of the excess carrier lifetimes can also be categorized into three parts labelled I to III as shown in Figure 4(b). The ranges of these regions are similar to the corresponding regions in Figure 4(a). The value of  $\tau_{\text{pt}}$  is less than the values of the SRH lifetimes  $\tau_{\text{nSRH}}$  and  $\tau_{\text{pSRH}}$  in region I of Figure 4(b) so that  $\tau_{\text{pt}}$  dominates the entire excess carrier lifetimes in the region. The values of  $\tau_{\text{pt}}$  and  $\tau_{\text{nt}}$  are extremely larger than the values of  $\tau_{\text{nSRH}}$  and  $\tau_{\text{pSRH}}$  near the recombination center  $E_{\text{SRH}}$ . Thus, both  $\tau_{\text{nSRH}}$  and  $\tau_{\text{pSRH}}$  dominate the entire excess carrier lifetimes near the recombination center  $E_{\text{SRH}}$ . In the first half of region II of Figure 4(b), the three excess carrier lifetimes  $\tau_{\text{nt}}$ ,  $\tau_{\text{nSRH}}$ , and  $\tau_{\text{pSRH}}$  dominate the entire energies between  $E_{\text{SRH}}$  and  $E_{1n}^{(1)}$ ; and only  $\tau_{\text{nt}}$  and  $\tau_{\text{nSRH}}$  dominate the entire energies between  $E_{1n}^{(1)}$  and  $E_n^{(m)}$ . In region III of Figure 4(b), the value of  $\tau_{\text{nt}}$  is less than that of the rest excess carrier lifetimes so that  $\tau_{\text{nt}}$  dominates the entire excess carrier lifetimes between  $E_n^{(m)}$  and the conduction band minimum  $E_C$ . The dominance of  $\tau_{\text{pt}}$  and  $\tau_{\text{nt}}$  over the other excess carrier lifetimes and that of  $U_{T\delta p}$  and  $U_{T\delta n}$  over  $U_{\text{SRH}}$  increase while traversing from the deep localized states toward the band edges as shown in Figures 4(a) and 4(b).

The dominance of  $U_{T\delta p}$  in the localized states below the recombination center ( $E_{\text{SRH}}$ ) and that of  $U_{T\delta n}$  in the localized states above  $E_{\text{SRH}}$  are in complete agreement with the report of several researchers on the existence of the dominant shallow and deep trap localized states in silicon [6–15, 20, 21, 26–31]. Localized states with energies closer to  $E_{\text{SRH}}$  (where  $\gamma_p = \gamma_n \rightarrow 0$ ) do not involve in the free

carriers' trapping and hence act as recombination centers. Since  $\gamma_p < 1$ , the values of the excess carrier lifetimes are not affected by the majority carrier trapping in the highly doped p-type semiconductor. Only minority carrier trapping ( $\gamma_n > 1$ ) at the localized states between energies  $E_{1n}^{(1)}$  and  $E_{2n}^{(1)}$  reduces the value of  $\tau_{nSRH}$  and increases the value of  $\tau_{ne}$  by a factor of  $1 + \gamma_n$  when compared to the values of  $\tau_{pSRH}$  and  $\tau_{pe}$ , respectively. This also approves the premise that only minority carrier traps and lifetimes play a very decisive role in controlling the performance of the semiconductor [1, 2, 15].

## 5. Conclusions

For the differentiation of the free-carrier trapping and recombination centers in the bandgap of p-type silicon, first, the expressions for the excess carrier accumulation rates at the localized states in the valence and the conduction bands are formulated. This enables us to describe the expressions for  $U_{SRH}$ ,  $U_{T\delta p}$ ,  $U_{E\delta p}$ ,  $U_{T\delta n}$ , and  $U_{E\delta n}$  and those of the corresponding excess carrier lifetimes. Upon applying the steady-state conditions to the rates of accumulations of excess carriers in the localized states and the valence and conduction bands, the expressions of  $\delta n$ ,  $\delta p$ ,  $\delta n_T$ , and  $\delta p_T$  are described. Next, the expressions for  $\gamma_p$  and  $\gamma_n$  are determined for the first time in this work. The steady-state values of the excess carrier lifetimes are also determined by dividing the steady-state values of the excess carrier densities in the bands and localized states by the corresponding transition rates.

The values of  $\gamma_n$  and  $\gamma_p$  are found to be very crucial and important in the categorization of the localized states in the bandgap as the recombination, the trapping, the acceptor, and the donor levels. The result applied to p-type silicon in this case investigates that  $U_{SRH}$  dominates the entire free-carrier transition rate at a very narrow deep localized state energy band far below the midgap and far above the Fermi energy level around the energy  $E_{SRH}$  that can be called as the recombination center. The hole trapping emission rate dominates the localized states between the valence band edge and the recombination center with the hole maximum trapping ratio very much closer to the Fermi energy level  $E_F$ . The electron trapping emission rate and  $U_{SRH}$  equally dominate the localized states between the recombination center and the energy level of maximum electron trapping ratio  $E_n^{(m)}$ . The entire localized states between  $E_n^{(m)}$  and the conduction band edge  $E_C$  are dominated by only the electron trapping emission rate.

Even though the excess carrier trapping emission rates are highly dominant in the very shallow localized state energy ranges closer to the band edges, the majority carrier (hole) trap has no effect on the excess carrier lifetime (performance) of the highly doped p-type semiconductor. Only very deep minority carrier (electron) traps with energies between  $E_{1n}^{(1)}$  and  $E_{2n}^{(1)}$  play vital roles in determining the performance of the semiconductor.

## Data Availability

The data supporting the findings of this study are available within the article and/or attached as the supplementary material.

## Conflicts of Interest

The author declares no conflicts of interest.

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