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Covariant Almost Analytic Vector Field On Q-Quasi Umbilical Hypersurface of a Trans Sasakian Manifold

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Authors' contributions

This work was carried out in collaboration between all authors. All authors designed the study, performed the theoretical study and analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Author MDS managed the analyses of the study and literature searches. All authors read and approved the final manuscript.

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Abstract

The present paper aims to study the axioms of covariant almost analytic vector field on Q-quasi umbilical hypersurface M of a trans-Sasakian manifold $\bar{M}$ with structure (ϕ, g, u, v, λ) and obtained the scalars a and b using 1-forms u, v covariant almost analytic for the hypersurface M to be totally umbilical and cylindrical.

Keywords: Trans-Sasakian manifolds; covariant almost analytic vector field; Q-quasi umbilical hypersurface.

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1 Introduction

The study of hypersurfaces with almost contact structures are important parts of differential geometry. It was 1967, M. Okumura [1] studied the totally umbilical hypersurfaces of a product Riemannian manifold and hypersurface of an almost r-paracontact Riemannian manifold studied by A. Bucki [2]. In 1970, S. I. Goldberg and K. Yano [3] studied the geometry of non-invariant hypersurfaces of almost contact manifolds. D. E. Blair [4] discussed the case of almost contact manifolds with Killing structure tensor. In 1989, D. Narain studied differential geometry of hypersurfaces with (f, g, u, v, λ)-structures [5]. Moreover, D. Narain and others also studied the non-invariant hypersurfaces of Sasakian, nearly Sasakian and para Sasakian manifold in (see [6], [7], [8]) respectively. After that R. Prasad and M. M. Tripathi [9] studied the transversal hypersurfaces of Kenmotsu manifolds. In 1985, J. A. Oubina introduced a new class of almost contact metric manifold known as trans-Sasakian manifold [10]. This class contains α-Sasakian and β-Kenmotsu manifold. Thus motivated by the studies referred above in the present paper, we have studied the covariant almost analytic vector filed on Q-quasi umbilical hypersurface of a trans-Sasakian manifold.

The present paper is organized as follows: In section 2, we give the brief introduction about the trans-Sasakian manifold. In section 3, the hypersurfaces of a trans-Sasakian manifold with (ϕ, g, u, v, λ)-structure have been discussed. In section 4, we express the Q-quasi umbilical hypersurface and in section 5, we obtained some results on covariant almost analytic vector field on Q-quasi umbilical hypersurface of a trans-Sasakian manifold.

2 Basic Results and Definitions

Let $\bar{M}$ be a $(2n + 1)$ dimensional manifold with almost contact metric structure $(\bar{\phi}, \xi, \eta, \bar{g})$, where $\bar{\phi}$ is a (1,1) tensor field, ξ is a vector field, η is a 1-form and $\bar{g}$ is a compatible Riemannian metric such that

$$\bar{\phi}^2 = -I + \eta \otimes \xi, \quad \phi\xi = 0, \quad \eta o \phi = 0, \quad \eta(\xi) = 1, \tag{2.1}$$

$$\bar{g}(\bar{\phi}X, \bar{\phi}Y) = \bar{g}(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$\bar{g}(X, \phi Y) = -\bar{g}(\phi X, Y), \quad \bar{g}(X, \xi) = \eta(X). \tag{2.3}$$

An almost contact metric structure $(\bar{\phi}, \xi, \eta, \bar{g})$, is called an trans Sasakian manifold if

$$(\bar{\nabla}_X \bar{\phi})(Y) = \alpha(\bar{g}(X, Y)\xi - \eta(Y)X) + \beta(\bar{g}(\phi X, Y)\xi - \eta(Y)\bar{\phi}X) \tag{2.4}$$

and it follows

$$\bar{\nabla}_X \xi = -\alpha\bar{\phi}X + \beta(X - \eta(X)\xi), \tag{2.5}$$

where $\bar{\nabla}$ denotes the Riemannian connection of the Riemannian metric $\bar{g}$, then the structure $(\bar{\phi}, \xi, \eta, \bar{g})$ is called a trans-Sasakian manifold.

If we define $G(X, Y) = \bar{g}(\phi X, Y)$, then we also find

$$G(X, Y) + G(Y, X) = 0, \tag{2.6}$$

$$G(X, \phi Y) = G(Y, \phi X), \tag{2.7}$$

$$G(\phi X, \phi Y) = G(X, Y). \tag{2.8}$$

3 Hypersurface of a Trans-Sasakian Manifold with (ϕ, g, u, v, λ)-structure

Let us consider a $2n$-dimensional manifold M embedded in $\bar{M}$ with embedding $i : M \rightarrow \bar{M}$. The map i induces a linear transformation map B, $B : T_p \rightarrow T_{ip}$. Let an affine unit normal N of M is in such way that $\bar{\phi}N$ is always tangent to the hypersurface and satisfying the linear transformations

$$\bar{\phi}BX = B\phi X + u(X)N, \tag{3.1}$$

$$\bar{\phi}N = -BU, \tag{3.2}$$

$$\xi = BV + \lambda N, \tag{3.3}$$

$$\eta(BX) = v(X), \tag{3.4}$$

where ϕ is $(1, 1)$ type tensor; U, V are vector fields; u, v are 1-form and λ is a C^∞-function. If $u \neq 0$, then M is called a non-invariant hypersurface of $\bar{M}$.

Operating (3.1), (3.2), (3.3) and (3.4) by $\bar{\phi}$ and using (2.1), (2.2), and (2.3) and taking tangent and normal parts separately, we get the following induced structures on M

$$\phi^2 X = -X + u(X)U + v(X)V, \tag{3.5}$$

$$u(\phi X) = \lambda v(X), \qquad v(\phi X) = -\eta(N)u(X), \tag{3.6}$$

$$\phi U = -\eta(N)V, \qquad \phi V = \lambda U, \tag{3.7}$$

$$u(U) = 1 - \lambda\eta(N), \qquad u(V) = 0, \tag{3.8}$$

$$v(U) = 0, \qquad v(V) = 1 - \lambda\eta(N) \tag{3.9}$$

and from (2.2) and (2.3), we get the induced metric g on M.

$$g(\phi X, \phi Y) = g(X, Y) - u(X)u(Y) - v(X)v(Y), \tag{3.10}$$

$$g(U, X) = u(X), \qquad g(V, X) = v(X). \tag{3.11}$$

If we consider $\eta(N) = \lambda$, we get the following structures on M

$$\phi^2 = -I + u \otimes U + v \otimes V, \tag{3.12}$$

$$\phi U = \lambda V, \qquad \phi V = \lambda U, \tag{3.13}$$

$$u \circ \phi = \lambda v, \qquad v \circ \phi = -\lambda u, \tag{3.14}$$

$$u(U) = 1 - \lambda^2, \qquad u(V) = 0, \tag{3.15}$$

$$v(U) = 0, \qquad v(V) = 1 - \lambda^2. \tag{3.16}$$

A manifold M with a metric g satisfying (3.10), (3.11) and (3.12) is called manifold with (ϕ, g, u, v, λ)-structure. Let ∇ be the induced connection on the hypersurface M of the affine connection $\bar{\nabla}$ of $\bar{M}$.

Now the Gauss and Weingarten equations are given respectively by

$$\bar{\nabla}_{BX}BY = B\nabla_X Y + h(X, Y)N, \tag{3.17}$$

$$\bar{\nabla}_{BX}N = BHX + w(X)N, \qquad \text{where } \; g(HY, Z) = h(Y, Z). \tag{3.18}$$

Here h and H are the second fundamental tensor of type $(0, 2)$ and $(1, 1)$ and w is a 1-form.

Now differentiating (3.1), (3.2), (3.3) and (3.4) covariantly and using (3.17), (3.18) and (2.4) and again re-using (3.1), (3.2), (3.3) and (3.4), we get

$$(\nabla_Y\phi)(X) = \alpha\{v(X)Y - g(X,Y)V\} + \beta\{\lambda g(\phi X,Y)V - v(Y)\phi X\} \tag{3.19}$$

$$-h(X,Y)U - u(X)HY,$$

$$(\nabla_Y u)(X) = \alpha\lambda g(X,Y) + \beta\{\lambda g(\phi X,Y) - u(X)v(Y)\} + g(X,\phi Y) - h(X,\phi Y), \tag{3.20}$$

$$(\nabla_Y v)(X) = \alpha g(\phi Y,X) - \beta\{\lambda g(Y,X) - u(X)u(Y) - v(X)v(Y)\} + \lambda h(X,Y), \tag{3.21}$$

$$\nabla_Y U = \phi HY + \beta w(Y)U - \alpha\lambda Y, \tag{3.22}$$

$$\nabla_Y V = \alpha\phi Y + \lambda HY, \tag{3.23}$$

$$h(Y,V) = \alpha u(Y) + \beta\lambda w(X), \tag{3.24}$$

$$h(Y,U) = u(HY) + w(X). \tag{3.25}$$

Since $h(X,Y) = g(HX,Y)$, then from (2.3) and (3.25), we get

$$h(Y,U) = 0 \Rightarrow HU = 0. \tag{3.26}$$

4 Q-Quasi Umbilical Hypersurface

M is called Quasi-umbilical hypersurface if

$$h(X,Y) = ag(X,Y) + bq(X)q(Y), \tag{4.1}$$

where a, b are scalar functions, q is a 1-form and if $g(Q,X) = q(X)$, where Q is a vector field, then M is called Q-quasi umbilical hypersurface. If $a = 0$, $b \neq 0$, then Q-quasi umbilical hypersurface M is called *cylindrical hypersurface*. If $a \neq 0$. $b = 0$, then Q-quasi-umbilical hypersurface M is called *totally umbilical* and if $a = 0$, $b = 0$, then Q-quasi umbilical hypersurface is *totally geodesic*. By using (4.1) in (3.19)-(3.25), we get the following:

$$(\nabla_Y\phi)(X) = \alpha\{v(X)Y - g(X,Y)V\} + \beta\{g(\phi X,Y)V - v(Y)\phi X\} \tag{4.2}$$

$$-\{ag(X,Y) + bq(X)q(Y)\}U - u(Y)\{aY + bq(Y)Q\},$$

$$(\nabla_Y u)(X) = \{(1+a)g(\phi X,Y)\} - \beta\{\lambda g(\phi X,Y) - u(X)w(Y)\} \tag{4.3}$$

$$+\alpha\lambda g(X,Y) + bq(X)q(Y),$$

$$(\nabla_Y v)(X) = \alpha g(\phi Y,X) - \beta\{\lambda g(Y,X) - u(X)u(Y) - v(X)v(Y)\} \tag{4.4}$$

$$+\lambda\{ag(X,Y) + bq(X)q(Y)\},$$

$$\nabla_Y U = \beta w(Y)U + \{a\phi Y + bq(Y)Q\} - \alpha\lambda Y, \tag{4.5}$$

$$\nabla_Y V = \alpha\phi Y + \lambda\{aY + bq(Y)Q\}, \tag{4.6}$$

$$h(Y,V) = ag(V,Y) + bq(V)q(Y), \tag{4.7}$$

$$|u(Q)|^2 = -\frac{a}{b}(1-\lambda^2) + w(X). \tag{4.8}$$

Also from (3.15), (3.24) and (4.1), we get

$$w(U) = \frac{\alpha(1-\lambda^2)}{\beta\lambda} \tag{4.9}$$

5 Covariant Almost Analytic Vector Field on Q-quasi Umbilical Hypersurface

1-form u and v are said to be covariant almost analytic if

$$u\left\{(\nabla_X\phi)(Y)-(\nabla_Y\phi)(X)\right\}=(\nabla_{\phi X}u)(Y)-(\nabla_X u)(\phi Y) \tag{5.1}$$

and

$$v\left\{(\nabla_X\phi)(Y)-(\nabla_Y\phi)(X)\right\}=(\nabla_{\phi X}v)(Y)-(\nabla_X v)(\phi Y). \tag{5.2}$$

Theorem 5.1. On Q-quasi umbilical hypersurface M with (ϕ, g, u, v, λ)-structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u be a covariant almost analytic, then we have

$$\begin{aligned}\alpha\left\{v(Y)u(X)-v(X)u(Y)\right\}=&-(2+2a+2\beta)g(\phi X,\phi Y)-2\alpha\lambda g(\phi X,Y)\\&-bq(\phi Y)q(\phi X)+\beta\left\{u(\phi Y)w(X)-u(Y)w(\phi X)\right\}\\&+2bq(X)u(Y)u(Q).\end{aligned} \tag{5.3}$$

Proof. From equation (4.2), we have

$$\begin{aligned}(\nabla_Y\phi)(X)=&\,\alpha\left\{v(X)Y-g(X,Y)V\right\}+\beta\left\{g(\phi X,Y)V-v(Y)\phi X\right\}\\&-\left\{ag(X,Y)+bq(X)q(Y)\right\}U-u(X)\left\{aY+bq(Y)Q\right\}\end{aligned}$$

and

$$\begin{aligned}(\nabla_X\phi)(Y)=&\,\alpha\left\{v(Y)X-g(Y,X)V\right\}+\beta\left\{g(\phi Y,X)V-v(X)\phi Y\right\}\\&-\left\{ag(Y,X)+bq(Y)q(X)\right\}U-u(Y)\left\{aX+bq(X)Q\right\}.\end{aligned}$$

Now from above equations we have

$$\begin{aligned}(\nabla_X\phi)(Y)-(\nabla_Y\phi)(X)=&\,\alpha\left\{v(Y)X-v(X)Y\right\}+\beta\left\{v(Y)\phi X-v(X)\phi Y\right\}\\&-u(Y)\left\{aX+bq(X)Q\right\}+u(X)\left\{aY+bq(Y)Q\right\}\end{aligned}$$

$$\begin{aligned}u\left\{(\nabla_X\phi)(Y)-(\nabla_Y\phi)(X)\right\}=&\,\alpha\left\{v(Y)X-v(X)Y\right\}\\&+b\left\{u(X)q(Y)-q(X)u(Y)\right\}u(Q).\end{aligned} \tag{5.4}$$

Also from equation (4.3)

$$\begin{aligned}(\nabla_X u)(Y)=&-\left\{(1+a)g(\phi Y,X)\right\}+\beta\left\{\lambda g(\phi Y,X)-u(Y)w(X)\right\}\\&+\alpha\lambda g(Y,X)+bq(\phi Y)q(X)\end{aligned}$$

replacing ϕ by ϕX and ϕY in the last equation, we find respectively

$$\begin{aligned}(\nabla_{\phi X}u)(Y)=&-\left\{(1+a)g(\phi Y,\phi X)\right\}+\beta\left\{\lambda g(\phi Y,\phi X)-u(Y)w(\phi X)\right\}\\&+\alpha\lambda g(Y,\phi X)+bq(\phi Y)q(\phi X)\end{aligned}$$

and

$$\begin{aligned}(\nabla_X u)(\phi Y)=&-(1+a)g(\phi X,\phi Y)+bq(Y)q(X)-bu(Y)q(U)q(X)\\&-\beta u(\phi Y)w(X)-\alpha\lambda g(X,\phi Y).\end{aligned}$$

From last two equations we find

$$\begin{aligned}(\nabla_{\phi X}u)(Y)-(\nabla_X u)(\phi Y)=&-(2+2a+2\beta)g(\phi X,\phi Y)-bq(\phi Y)q(\phi X)\\&+bu(Y)q(U)q(X)+\beta\left\{u(\phi Y)w(X)-u(Y)w(\phi X)\right\}\\&-bq(Y)q(X)+\alpha\lambda g(X,\phi Y)-\alpha\lambda g(\phi X,Y)\end{aligned} \tag{5.5}$$

and using equation (5.1) in (5.4) and (5.5), we get (5.3). □

Corollary 5.1. On Q-quasi umbilical hypersurface M with (ϕ, g, u, v, λ)-structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u be a covariant almost analytic, then we also have

$$2\alpha\lambda^2 = 0, \quad \alpha \neq 0, \tag{5.6}$$

where λ is covariant constant along $\bar{M}$.

Proof.

$$\alpha(1-\lambda^2)^2 = -(2+2a+2\beta).0 + 2\alpha\lambda^2(1-\lambda^2) + \beta\lambda(1-\lambda^2)w(U)$$
$$\alpha(1-\lambda^2) = +2\alpha\lambda^2(1-\lambda^2) + \beta\lambda(1-\lambda^2)w(U)$$
$$\alpha(1-\lambda^2) = +2\alpha\lambda^2(1-\lambda^2) + \beta\lambda\left\{\frac{\alpha(1-\lambda^2)}{\beta\lambda}\right\}$$
$$\alpha(1-\lambda^2) = 2\alpha\lambda^2 + \alpha(1-\lambda^2)$$
$$2\alpha\lambda^2 = 0, \qquad \alpha \neq 0,$$

which shows that λ is covariant constant along $\bar{M}$.

□

Theorem 5.2. On cylindrical hypersurface M with (ϕ, g, u, v, λ)-structure of a trans-Sasakian manifold $\bar{M}$ with covariant analytic vector fields U and V, we have

$$\alpha\{v(Y)u(X) - v(X)u(Y)\} = -(2+2\beta)g(\phi X, \phi Y) - 2\alpha\lambda g(\phi X, Y) - bq(\phi Y)q(\phi X) + \beta\{u(\phi Y)w(X) - u(Y)w(\phi X)\}. \tag{5.7}$$

Proof. Putting $a = 0$ in equation (5.3) and using $u(Q) = 0$, we get equation (5.7).

□

Corollary 5.2. On cylindrical hypersurface M with (ϕ, g, u, v, λ)-structure of a trans-Sasakian manifold $\bar{M}$ with covariant analytic vector fields U, V, we have

$$\alpha u(Y) + (1+\beta)v(Y) = 0, \tag{5.8}$$

$$q(Q) = 0. \tag{5.9}$$

Proof. Putting $X = V$ in equation(5.7), we get

$$-\alpha(1-\lambda^2)u(Y) = -(2+2\beta)g(\phi V, \phi Y) - 2\alpha\lambda g(\phi V, Y) - bq(\phi Y)q(\phi V) + \beta\{u(\phi Y)w(V) - u(Y)w(\phi V)\}$$
$$-\alpha(1-\lambda^2)u(Y) = -(2+2\beta)v(Y)\lambda^2 - 2\alpha\lambda^2 u(Y) + \beta\{u(\phi Y)w(V) - u(Y)w(\phi V)\}$$
$$-\alpha(1-\lambda^2)u(Y) = -(2+2\beta)v(Y)\lambda^2 - 2\alpha\lambda^2 u(Y) + \beta u(\phi Y)w(V) - \beta\lambda u(Y)w(U)$$
$$\alpha(3\lambda^2 - 1)u(Y) = -\beta\lambda u(Y)\left\{\frac{\alpha(1-\lambda^2)}{\beta\lambda}\right\} - (2+2\beta)v(Y)\lambda^2$$
$$2\alpha\lambda^2 u(Y) = -(2+2\beta)v(Y)\lambda^2$$
$$\alpha u(Y) + (1+\beta)v(Y) = 0.$$

Also from equation (5.7), we get

$$u(X)V - v(X)U = -(2+2\beta)\phi X - 2\alpha\lambda\phi X - b\phi Q q(\phi X) - bq(X)Q + \beta\{w(X)\phi U - w(\phi X)U\}$$

which on contracting with respect to X gives

$$0 = -bq(\phi^2 Q) - bq(Q) + \beta \{\lambda w(X) - \lambda w(V)\}$$

or

$$bq(Q) = (V\lambda) = 0,$$

since $b \neq 0$, so $q(Q) = 0$. □

Theorem 5.3. On Q-quasi umbilical hypersurface M with (ϕ, g, u, v, λ)-structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u is covariant almost analytic and M is totally umbilical, then we have

$$\alpha \{v(Y)u(X) - v(X)u(Y)\} = -(2 + 2a + 2\beta)g(\phi X, \phi Y) - 2\alpha\lambda g(\phi X, Y) + \beta \{u(\phi Y)w(X) - u(Y)w(\phi X)\} \tag{5.10}$$

Proof. Putting $b = 0$ in equation (5.3), we get equation (5.10). □

Corollary 5.3. On Q-quasi umbilical hypersurface M with (ϕ, g, u, v, λ)-structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u is covariant almost analytic and M is totally umbilical, then we have

$$a = \left\{ \frac{\alpha(1 - 3\lambda^2)v(Y)}{\lambda} \right\} - \left\{ \frac{\alpha(1 - 3\lambda^2)}{\lambda} + 2(1 + \beta) \right\} u(Y) \tag{5.11}$$

Proof. Put $X = U$ in equation (5.10), we get

$$\alpha(1 - \lambda^2)v(Y) = -(2 + 2a + 2\beta)v(\phi Y) + 2\alpha\lambda^2 v(Y) + \beta \{u(\phi U)w(X) - u(Y)w(\phi U)\}$$

$$\alpha(1 - \lambda^2)v(Y) = -(2 + 2a + 2\beta)v(\phi Y) + 2\alpha\lambda^2 v(Y) + \beta \left\{ u(\phi Y)\frac{\alpha(1 - \lambda^2)}{\beta\lambda} \right\}$$

$$\alpha(1 - \lambda^2)v(Y) - 2\alpha\lambda^2 v(Y) = -(2 + 2a + 2\beta)u(Y) + u(Y) \left\{ u(\phi Y)\frac{\alpha(1 - \lambda^2)}{\lambda} \right\}$$

$$\alpha(1 - 3\lambda^2)v(Y) = u(Y) \left\{-2\lambda - 2a\lambda - 2\beta\lambda + \alpha(1 - \lambda^2)\right\}$$

$$\alpha(1 - 3\lambda^2)v(Y) - \alpha(1 - \lambda^2)u(Y) = -2u(Y) \{\lambda + a\lambda + \beta\lambda\}$$

$$a\lambda = \alpha(1 - 3\lambda^2)V - \alpha(1 - \lambda^2)U + 2U\lambda + 2\beta\lambda U$$

which on further solving gives (5.11). □

Theorem 5.4. On Q-quasi umbilical hypersurface M with (ϕ, g, u, v, λ)-structure of a trans-Sasakian manifold $\bar{M}$, if 1-form v is covariant almost analytic, then we have

$$\alpha \{u(X)v(Y) - u(Y)v(X)\} = -2\alpha g(\phi X, \phi Y) + 2(\lambda + \beta)ag(\phi X, Y) + \lambda b \{q(\phi X)q(Y) - q(X)q(\phi Y)\} \tag{5.12}$$

Proof. From equation (4.2), we get

$$(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X) = \alpha \{v(Y)X - v(X)Y\} + \beta \{v(Y)\phi X - v(X)\phi Y\} - u(Y) \{aX + bq(X)Q\} + u(X) \{aY + bq(Y)Q\}$$

$$(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X) = \beta \{v(Y)\phi X - v(X)\phi Y\} + a \{u(X)v(Y) - u(Y)v(X)\} \tag{5.13}$$

From (4.3), we have

$$(\nabla_{\phi X} v)(Y) = (\lambda a + \beta)g(\phi X, Y) + \lambda \{u(X)v(Y) - v(X)u(Y)\}$$

and

$$(\nabla_X v)(\phi Y) = -\alpha \{g(X,Y) - u(X)u(Y) - v(X)v(Y)\} + (\lambda - \beta)g(X, \phi Y)$$
$$+ \lambda \{v(X)u(Y) - u(X)v(Y)\}.$$

Therefore we have

$$(\nabla_{\phi X} v)(Y) - (\nabla_X v)(\phi Y) = -2\alpha g(\phi X, \phi Y) + 2(\lambda + \beta) a g(\phi X, Y) \quad (5.14)$$
$$+ \lambda b \{q(\phi X)q(Y) - q(X)q(\phi Y)\}.$$

By using equation (5.2) in (5.13) and (5.14), we get (5.12).

□

Theorem 5.5. Let the 1-from v be a covariant almost analytic on Q-quasi umbilical hypersurface M with (ϕ, g, u, v, λ)-structure of a trans-Sasakian manifold $\bar{M}$ if it is also cylindrical, then we have

$$2g(\phi X, \phi Y) = \lambda b \{q(\phi X)q(Y) - q(X)q(\phi Y)\} \quad (5.15)$$
$$- \beta \{v(Y)v(\phi X) - v(X)v(\phi Y)\}$$

with $X \neq U$.

Proof. Putting $\alpha = 0$ in equation (5.12), we obtain equation (5.15). Further $X = U$, then from (5.15), we have

$$2g(\phi U, \phi Y) = \lambda b \{-\lambda q(V)q(Y) - q(U)q(\phi Y)\}$$
$$- \beta \{v(Y)v(\phi U)\}$$
$$-2\lambda g(V, \phi Y) + \beta g(V, Y)g(V, \phi U) = 0$$

or

$$-2\lambda v(\phi Y) + \beta v(Y)V(\phi U) = 0,$$

since $v(\phi Y) \neq 0$ and $\lambda \neq 0$, therefore $X \neq U$. □

Theorem 5.6. Let the 1-form v be a covariant almost analytic on Q-quasi umbilical non-invariant hypersurface of M with (ϕ, g, u, v, λ)-structure of a trans-Sasakian manifold $\bar{M}$, if it is also totally umbilical, then we have

$$\alpha \{u(X)v(Y) - v(X)u(Y)\} = -2\alpha g(\phi X, \phi Y) + 2(\lambda + \beta) a g(\phi X, Y) \quad (5.16)$$
$$- \beta \{v(Y)v(\phi X) - v(X)v(\phi Y)\},$$

with $X \neq U$.

Proof. Putting $b = 0$ in equation (5.12), we get (5.14). If we take $X = U$ in(5.16), we get

$$a(1 - \lambda^2)v(Y) + 2(\lambda + \beta) a v(Y) = 2\alpha\lambda v(\phi Y) - \beta v(Y)u(\phi U)$$
$$a \left\{1 - \lambda^2 + 2(\lambda + \beta) + \beta u(\phi U)\right\} v(Y) = -2\alpha\lambda v(\phi Y)$$
$$a \left\{\lambda^2 - 2(\lambda + \beta) - \beta U - 1\right\} v(Y) = -2\alpha\lambda^2 U$$

i.e. V and U are linearly dependent, which is a contradiction. Thus $X \neq U$. □

6 Conclusion

In the present paper we have studied the axioms of covarinat almost analytic vector field on Q-quasi umbilical hypersurface of a trans-Sasakian manifold. We prove that Q-quasi umbilical hypersurface M with (ϕ, g, u, v, λ) and M is totally umbilical also totally geodesic are very strong assumptions they are correlated with Yang-Mills theory.

Competing Interests

Authors have declared that no competing interests exist.

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