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A Stochastic Model of the Dynamics of Stock Price for Forecasting

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

In this work, a stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE) is formulated. We considered four different stocks and their market prices. The likelihood of each change occurring in the stock prices was noted, the drift (the expectation) and the volatility (the covariance) of the change were computed leading to the formulation of stochastic differential equations. Changes in the prices of the stocks were studied for an average of 60 days. The drift and the volatility coefficients for the stochastic differential equations were determined and the Euler-Maruyama method for system of stochastic differential equations was used to simulate the stock prices. With the aid of the simulation we carried out a fore-cast of the prices of the stocks for a short time interval. A consideration of the different stock prices over a period of forty months, stock S1 seems to give the best return on investment compared with stocks S2, S3 and S4. The investor after observing the trend over longer period can invest in the stock that will yield the best returns. Our analysis enables us to compare as many as four stocks in order to advise the investor on where best to make investment.

Keywords: Stochastic model; drift and volatility coefficients; Wiener process; Euler- Maruyama.

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1 Introduction

The stock market has become an essential market playing vital roles in economic prosperity by fostering capital formation and sustaining economic growth in most economies across the world. Stock markets are more than a place to trade securities; they operate as facilitator between savers and users of capital by means of pooling funds, sharing risk and transferring wealth. Stock markets are essential for economic growth as they ensure the flow of resources to the most productive investment opportunities, [1].

In 12th century France, the *courretiers de change* were concerned with managing and regulating the debts of agricultural communities on behalf of the banks. These men could be called the first brokers because they also traded with debts. In the middle of the 13th century, the Venetian bankers began to trade in government securities. There are now stock markets in virtually every developed and most developing economy. The stock market is one of the most important ways for companies to raise money. This allows businesses to be publicly traded, and raise additional financial capital for expansion by selling shares of ownership of the company in a public market. The liquidity that an exchange affords the investors enables their holders to quickly and easily sell securities. This is an attractive feature of investing in stocks, compared to other less liquid investments such as property and other assets. History has shown that the price of stocks and other assets is an important part of the dynamics of economic activity, and can influence or be an indicator of social mood, [2].

Expounding on the role of an efficient stock market, a point was made thus: “There is a general consensus that when financial markets are very competitive and efficient, prices quickly reflect all the available information. There is also a widespread belief that competitive and efficient markets enable the efficient allocation of scarce capital among alternative investment opportunities”, [3]. Also the primary role of the capital market is allocation of ownership of the economy’s capital stock. In general terms, the ideal is a market in which prices provide accurate signals for resource allocation: that is, a market in which firms can make production – investment decisions, and investors can choose among the securities that represents ownership of firm’s activities under the assumption that security prices at any time “fully reflect” all available information, [4].

Predicting stock and stock price index is difficult due to the uncertainties involved, [5]. It is worth noting that stock price all over the world including Nigeria is characterized by upward and downward movements. The known Efficient Market Theory (EMT) believes that stock prices reflect everything that is known about a company and hence can be predicted based on fundamental analysis, while proponents of technical analysis attempt at forecasting future security prices based on historical data, [6].

The application of financial engineering principles in investment management has changed the landscape of financial modeling so that computationally intensive methods such as Monte Carlo simulations and simulation of sample paths of stochastic differential equations are now widely used to represent variation in stock prices. Powerful specialized mathematical languages and vast statistical software libraries have also been developed over the years. The ability to program sequences of statistical operations within a single programming language has been a big step forward, [7].

Financial analysts who invest in stock markets are usually not aware of the stock market behavior. They face the problem of stock trading, not knowing which stocks to buy and which to sell in order to gain more profits. Both financial analysts and prospective investors require daily information in predicting the behaviour of stock prices in Nigeria, [8].

In [9], the work indicated that there are reasons that the random walk behavior of stock prices should hold. There is evidence suggesting that stock prices do follow a random walk. According to [10] Stock prices could be determined by micro and macro-economic factors. These factors include book value of the firm, dividend per share, earning per share (EPS), price-earnings ratio and dividend cover, [11]. There is an argument that stock prices of a firm are influenced by firm beta ratios, that is, its market value to book value [12]. This argument in [12] was challenged in [13]. They argued that the stochastic discount factor and future payoffs determine the stock prices. Uncertainty and asymmetric information have been identified as strong influences of firms stock pricing which lead to under price, [14]. Also an argument in [9] exists, which states that stock prices can be viewed as predictions of investors earnings, therefore, it is reasonable that the variation in prices should be no greater than variation in firm EPS.

Using cash flow valuation model, an increase in expected inflation rate is likely to lead to economic tightening policies that would have negative effect upon stock prices. According to [15], a rise in the rate of inflation increases the nominal risk free rate and raises the discount rate. However, [16] argued that the cash flows does not rise at the same rate as inflation, and the rise in discount rate leads to lower stock prices, [6]. A report by dynamic portfolio limited shows that if the inflation rate is high, the tendency is that as the real income declines, the investors end up selling their assets, including stocks to enhance their purchasing power. The reverse is the case when the inflation rate is low, investors would like to acquire more assets. In essence, the era of high inflation rate negatively affects stock prices while low inflation rate boost stock prices.

On the effect of monetary policy variables on stock price moments, an examination of the relation between stock prices and inflation in the Nigerian stock market shows that stock prices are strongly driven by the level of economic activity measured by GDP, interest rate, money stock, and financial deregulation. On the other hand, the findings of the study show that oil price volatility has no significant effect on stock prices. A rise in interest rate may encourage investors to switch from the stock market to the money market. Reduced interest rate encourages demand for cash for speculative purpose and therefore may boost stock market activities. There is a relationship between bond yield, the level of stock prices and the price earnings ratio. The lower the yield on debt instruments, the higher the stock prices as well as the price earnings ratio. On the other hand the higher the yield on bonds, the lower the stock prices, [1].

An examination on the relationship between stock market capitalization and interest rate, using an ordinary regression analysis showed that the prevailing interest rate exerts positive influence on stock market capitalization. Exchange rate is another factor pointed out in the literature as a key determinant of stock price movements, [17]. The instability of exchange rate can cause speculation in foreign exchange market; disrupt international credit operations and international stock market operations. The instability can also lead to crisis of confidence that could cause capital flight, or a large-scale withdrawal of short-term credit facilities. If there is high exchange rate it would encourage round tripping and discourage stock market investment. It will cause operating cost upward movement and lower corporate profit in the real sector: The higher the operating cost the lower the profit. When the value of the currency is dropping, the incentive to invest by foreign investors in the domestic economy is lost. This can affect the stock market and stock prices, [18].

Johansen multivariate co-integration test and innovation accounting techniques from Vector Error Correction Model (VECM) was employed to study the effect of macroeconomic variables and stock price movements in Ghana, using quarterly data covering the period from the first quarter 1991 to the last quarter 2006. It was established that there is co-integration between macroeconomic variables identified and Stock prices in Ghana, indicating long run relationship. The result also shows that interest rate is the key determinant of the share price movements in Ghana. Money supply and country GDP are also seen as potential determinants of stock price movements. Contraction in money stock is expected to have a negative impact on stock prices, while an upward movement in GDP could raise stock prices due to the potential for higher profits arising from a healthy business climate. However, when the GDP is on the downward trend, there is likelihood of stock prices dropping, [18].

A test was conducted on the long run relationship between stock prices and changes in real macroeconomic activity in the Australian stock market in the period 1960 to 1998. The real macroeconomic activities include real GDP, real private consumption, real money supply, and real oil price. The results of the study indicated that there is a long run relationship between stock prices and real macroeconomic activity, [19].

According to [20], an examination on the Nigerian Stock Market performance and the Nigerian economic activity from 1985 to 2005 was conducted and the study concluded that among the variables examined in the vector-autoregressive (VAR) model, the price of the Nigerian crude oil, exchange rate and the role of inflation are the most significant macroeconomic variables influencing the aggregate stock market returns in Nigeria.

In [21], an examination was conducted on the relevance of the Nigerian crude oil price in the evaluation of the long-run performance of the Nigerian Stock Market. The study concluded that the Nigerian Stock Market and the oil price are tied together in the long-run as a rise in the price of oil leads to a decline in the return of performance of the stock market.

[21] reports that the oil price increase creates an indirect effect on the stock price through several ways among which are: (i) making the consumers source for alternative energies, (ii) increasing risk and uncertainty which negatively affects the stock price and reduces wealth and investment.

The mathematical development of present-day economic and finance theory began in Lausanne, Switzerland at the end of the nineteenth century, with the development of the mathematical equilibrium theory by Leon Walras and Wilfredo Pareto. At the beginning of the twentieth century, Louis Bachelier in Paris and Filip Lundberg in Uppsala (Sweden) developed sophisticated mathematical tools to describe uncertain price and risk processes. Since then, observations of prices of stocks, has been modeled as a stochastic process. A stochastic process with state space S is a collection of random variables $\{X_t, \quad t \in T\}$ defined on the same probability space (Ω, f, P) evolving over time. The set T is called its parameter set. If $T = \{0,1,2,...\}$, the process is said to be of a discrete parameter set. If T is not countable, the process is said to have a continuous parameter set. The index t represents time, and X_t as the "state" or the "position" of the process at time t. If the state space is R in most usual examples, then the process is said to be real-valued.

The first quantitative work on Brownian motion was introduced in 1900 by the French Mathematician Bachelier who used it in his dissertation to model the price movements of stocks and commodities, [22]. Variation in stock prices and other phenomenon involving uncertainties when modeled as a stochastic process, leads to Stochastic Differential Equations (SDE). For example, in finance and insurance, the concept of cooperate defaults, operational failures, insured accidents, uncertainties in foreign exchange and prices of stocks can be naturally modeled by Stochastic Differential equations (SDEs), [23]. Stochastic differential equations with jumps were used for modeling credit events like defaults and credit rating changes, stochastic differential equations could be applied to modeling of short time rate typically set by the central banks, [24]. Models for the dynamics of financial quantities specified by SDEs have become increasing popular in the recent past.

In [25] the binomial Asset Pricing model looked at the price of assets as going up or down by a factor u or d respectively. This model however does not take cognizance of the possibility of the asset price remaining stable within the time interval considered. In [26] a method was proposed for formulating stochastic models based on observation of the relevant parameters rather than the assumption that the drift and the volatility coefficient are linear functions of the solutions. Extending Allen's model, three selected stocks were considered for thirty days, the relevant parameters were observed and the drift and volatility were obtained which formed the coefficients of the stochastic differential equations, [27]. The model formulated in this work is an extension of [27]. We want to extend the works of [27] by considering four stocks. Though the calculations were longer and more complex, the result can help researchers have more insight in stock exchange.

Table 2.1. Daily prices in naira of four selected stocks for 60 days

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S_1	20.71	21.60	20.63	19.65	21.66	22.80	22.87	20.83	18.90	18.00	21.49	21.49	21.49	21.49	21.49
S_2	22.52	22.52	22.52	22.52	22.52	22.52	22.52	22.52	22.52	22.52	21.40	22.52	22.52	22.52	22.52
S_3	21.02	20.02	20.00	19.32	19.32	21.40	21.40	21.40	21.40	20.39	20.47	19.50	19.95	19.95	19.95
S_4	21.01	21.52	21.52	21.52	20.00	20.00	20.00	19.52	18.50	17.27	19.90	19.90	19.90	19.90	22.00

Day	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
S_1	21.49	21.49	21.70	20.71	20.71	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.42
S_2	22.52	22.52	22.52	22.52	22.52	20.02	20.99	20.99	20.99	21.00	20.38	20.38	20.38	20.38	20.38
S_3	19.95	20.57	20.57	20.57	20.57	21.50	21.50	21.50	21.00	21.00	20.47	20.47	20.47	20.47	20.47
S_4	22.00	20.00	19.20	19.20	21.01	19.90	19.90	19.90	19.90	19.90	19.90	19.90	19.90	19.90	19.90

Day	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
S_1	19.01	19.01	20.01	20.01	20.01	20.01	20.01	20.01	20.01	20.00	19.50	19.01	19.01	19.07	19.07
S_2	20.50	20.50	20.10	20.10	20.10	20.02	20.02	20.02	20.02	20.02	21.35	21.35	21.35	20.50	20.50
S_3	21.50	21.50	21.50	21.50	21.50	21.57	21.57	21.57	21.57	21.50	22.60	22.60	22.60	21.50	22.00
S_4	19.90	19.90	19.90	19.90	19.90	19.90	19.90	19.90	20.00	20.00	15.03	16.50	17.00	17.00	17.00

Day	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
S_1	19.8	19.44	18.52	19.52	19.01	18.99	19	19	19	19	19.5	19.5	19.5	19.5	19.5
S_2	20.5	20.5	20.5	20.5	20.5	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35
S_3	21.5	21.5	21.5	21.5	21.5	21.05	22.15	22.15	22.15	22.15	22.15	21.1	21.5	22.5	22.5
S_4	17	17.57	17.91	19.9	19.9	15.05	15.05	15.05	15.05	15.05	15.05	15.05	15.05	15.05	15.05

S_1 represents INTERBREW, S_2 represents AP, S_3 represents ASHAKACEM while S_4 represents STANBIC.
Source: [28]

2 Data Presentation

Stock prices are published daily. In other to characterize the drifts and volatilities which will form the coefficients of the stochastic differential equations, the daily stock prices of four selected stocks were observed for sixty days. The Table 2.1 shows the daily prices of the selected stocks for the chosen days.

3 Model Formulation

In order to develop a stochastic model that captures the dynamics of changes in the prices of stocks, an approach that is analogous to the method used over the years to formulate models resulting in deterministic ordinary differential equation is used. This involves studying the dynamics of a system of interest for a short time interval Δt after which the information obtained from the short time study of the dynamics of the system were used to formulate the mathematical model of the system. A stochastic system studied for a discrete time interval, results to a discrete stochastic model and for a sufficiently small Δt, the discrete time stochastic model leads to stochastic differential equation model as $\Delta t \to 0$, [26]. This approach is different from the commonly used approach that is based on the hypothesis that the drift and diffusion coefficient are linear function of the solution.

3.1 Stochastic model of dynamics of change in four stock prices

Consider four stocks S_1, S_2, S_3 and S_4 subjected to random influence by the market forces. We assume that in a small interval of time Δt, a stock price may change by losing one unit (-1), remaining stable (0) or gaining one unit (+1).

There are $3^4 = 81$ possibilities by which the stocks S_1, S_2, S_3 and S_4 may vary in the small time interval Δt. These possibilities are indicated in Table 3.1 below.

Table 3.1. Possible outcome of change in 4 stock prices and their probabilities

S/N	Change in Stock price $\Delta S = \left[\Delta S_1, \Delta S_2, \Delta S_3, \Delta S_4\right]^T$	Probabilities
1	[-1,0,0,0]	$p_1 = d_1 S_1$
2	[0,-1,0,0]	$p_2 = d_2 S_2$
3	[0,0,-1,0]	$p_3 = d_3 S_3$
4	[0,0,0,-1]	$p_4 = d_4 S_4$
5	[1,0,0,0]	$p_5 = b_1 S_1$
6	[0,1,0,0]	$p_6 = b_2 S_2$
7	[0,0,1,0]	$p_7 = b_3 S_3$
8	[0,0,0,1]	$p_8 = b_4 S_4$
9	[-1,-1,0,0]	$p_9 = \alpha_{11} S_1 S_2$

S/N	**Change in Stock price** $\Delta S=\left[\Delta S_1,\Delta S_2,\Delta S_3,\Delta S_4\right]^T$	**Probabilities**
10	[-1,1,0,0]	$p_{10}=\alpha_{12}S_1S_2$
11	[1,-1,0,0]	$p_{11}=\alpha_{21}S_1S_2$
12	[1,1,0,0]	$p_{12}=\alpha_{22}S_1S_2$
13	[-1,0,-1,0]	$p_{13}=\beta_{11}S_1S_3$
14	[-1,0,1,0]	$p_{14}=\beta_{12}S_1S_3$
15	[1,0,-1,0]	$p_{15}=\beta_{21}S_1S_3$
16	[1,0,1,0]	$p_{16}=\beta_{22}S_1S_3$
17	[-1,0,0,-1]	$p_{17}=\gamma_{11}S_1S_4$
18	[-1,0,0,1]	$p_{18}=\gamma_{12}S_1S_4$
19	[1,0,0,-1]	$p_{19}=\gamma_{21}S_1S_4$
20	[1,0,0,1]	$p_{20}=\gamma_{22}S_1S_4$
21	[0,-1,-1,0]	$p_{21}=\eta_{11}S_2S_3$
22	[0,-1,1,0]	$p_{22}=\eta_{12}S_2S_3$
23	[0,1,-1,0]	$p_{23}=\eta_{21}S_2S_3$
24	[0,1,1,0]	$p_{24}=\eta_{22}S_2S_3$
25	[0,-1,0,-1]	$p_{25}=\delta_{11}S_2S_4$
26	[0,-1,0,1]	$p_{26}=\delta_{12}S_2S_4$
27	[0,1,0,-1]	$p_{27}=\delta_{21}S_2S_4$
28	[0,1,0,1]	$p_{28}=\delta_{22}S_2S_4$
29	[0,0,-1,-1]	$p_{29}=\xi_{11}S_3S_4$
30	[0,0,-1,1]	$p_{30}=\xi_{12}S_3S_4$
31	[0,0,1,-1]	$p_{31}=\xi_{21}S_3S_4$
32	[0,0,1,1]	$p_{32}=\xi_{22}S_3S_4$
33	[-1,-1,-1,0]	$p_{33}=\varphi_{111}S_1S_2S_3$
34	[-1,-1,1,0]	$p_{34}=\varphi_{112}S_1S_2S_3$
35	[-1,1,-1,0]	$p_{35}=\varphi_{121}S_1S_2S_3$
36	[-1,1,1,0]	$p_{36}=\varphi_{122}S_1S_2S_3$

S/N	Change in Stock price $\Delta S=\left[\Delta S_1,\Delta S_2,\Delta S_3,\Delta S_4\right]^T$	Probabilities
37	[1,-1,-1,0]	$p_{37}=\varphi_{211}S_1S_2S_3$
38	[1,-1,1,0]	$p_{38}=\varphi_{212}S_1S_2S_3$
39	[1,1,-1,0]	$p_{39}=\varphi_{221}S_1S_2S_3$
40	[1,1,1,0]	$p_{40}=\varphi_{222}S_1S_2S_3$
41	[-1,-1,0,-1]	$p_{41}=\theta_{111}S_1S_2S_4$
42	[-1,-1,0,1]	$p_{42}=\theta_{112}S_1S_2S_4$
43	[-1,1,0,-1	$p_{43}=\theta_{121}S_1S_2S_4$
44	[-1,1,,0,1]	$p_{44}=\theta_{122}S_1S_2S_4$
45	[1,-1,0,-1]	$p_{45}=\theta_{211}S_1S_2S_4$
46	[1,-1,0,1]	$p_{46}=\theta_{212}S_1S_2S_4$
47	[1,1,0,-1]	$p_{47}=\theta_{221}S_1S_2S_4$
48	[1,1,0,1]	$p_{48}=\theta_{222}S_1S_2S_4$
49	[-1,0,-1,-1]	$p_{49}=\lambda_{111}S_1S_3S_4$
50	[-1,0,-1,1]	$p_{50}=\lambda_{112}S_1S_3S_4$
51	[-1,0,1,-1]	$p_{51}=\lambda_{121}S_1S_3S_4$
52	[-1,0,1,1]	$p_{52}=\lambda_{122}S_1S_3S_4$
53	[1,0,-1,-1]	$p_{53}=\lambda_{211}S_1S_3S_4$
54	[1,0,-1,1]	$p_{54}=\lambda_{212}S_1S_3S_4$
55	[1,0,1,-1]	$p_{55}=\lambda_{221}S_1S_3S_4$
56	[1,0,1,1]	$p_{56}=\lambda_{222}S_1S_3S_4$
57	[0,-1,-1,-1]	$p_{57}=\psi_{111}S_2S_3S_4$
58	[0,-1,-1,1]	$p_{58}=\psi_{112}S_2S_3S_4$
59	[0,-1,1,-1]	$p_{59}=\psi_{121}S_2S_3S_4$
60	[0,-1,1,1]	$p_{60}=\psi_{122}S_2S_3S_4$
61	[0,1,-1,-1]	$p_{61}=\psi_{211}S_2S_3S_4$
62	[0,1,-1,1]	$p_{62}=\psi_{212}S_2S_3S_4$
63	[0,1,1,-1]	$p_{63}=\psi_{221}S_2S_3S_4$

S/N	**Change in Stock price** $\Delta S=\left[\Delta S_1,\Delta S_2,\Delta S_3,\Delta S_4\right]^T$	**Probabilities**
64	[0,1,1,1]	$p_{64}=\psi_{222}S_2S_3S_4$
65	[-1,-1,-1,-1]	$p_{65}=\pi_{1111}S_1S_2S_3S_4$
66	[-1,-1,-1,1]	$p_{66}=\pi_{1112}S_1S_2S_3S_4$
67	[-1,-1,1,-1]	$p_{67}=\pi_{1121}S_1S_2S_3S_4$
68	[-1,-1,1,1]	$p_{68}=\pi_{1122}S_1S_2S_3S_4$
69	[-1,1,-1,-1]	$p_{69}=\pi_{1211}S_1S_2S_3S_4$
70	[-1,1,-1,1]	$p_{70}=\pi_{1212}S_1S_2S_3S_4$
71	[-1,1,1,-1]	$p_{71}=\pi_{1221}S_1S_2S_3S_4$
72	[-1,1,1,1]	$p_{72}=\pi_{1222}S_1S_2S_3S_4$
73	[1,-1,-1,-1]	$p_{73}=\pi_{2111}S_1S_2S_3S_4$
74	[1,-1,-1,1]	$p_{74}=\pi_{2112}S_1S_2S_3S_4$
75	[1,-1,1,-1]	$p_{75}=\pi_{2121}S_1S_2S_3S_4$
76	[1,-1,1,1]	$p_{76}=\pi_{2122}S_1S_2S_3S_4$
77	[1,1,-1,-1]	$p_{77}=\pi_{2211}S_1S_2S_3S_4$
78	[1,1,-1,1]	$p_{78}=\pi_{2212}S_1S_2S_3S_4$
79	[1,1,1,-1]	$p_{79}=\pi_{2221}S_1S_2S_3S_4$
80	[1,1,1,1]	$p_{80}=\pi_{2222}S_1S_2S_3S_4$
81	[0,0,0,0]	$p_{81}=1-\sum_{j=1}^{80}p_i$

Here, ΔS represents change in stock price. For example, $\Delta S=[1,0,0,0]$ represent a gain of 1 unit in stock S_1 while stocks S_2,S_3 and S_4 remain stable; $\Delta S=[1,-1,1,-1]$ represents a simultaneous gain of 1 unit by stocks S_1 and S_3 and a simultaneous loss of 1 unit by stocks S_2 and S_4. It is assumed that the change in the stock price is proportional to the price of the stock. For simultaneous gains/losses, we assume that the probability of the change is proportional to the product of the stock prices. This is a reasonable assumption as, supposing that the one of the stock price is zero then, the probability of a simultaneous gain is zero. It is also assumed that Δt is sufficiently small so that p_{81} which is the probability that there is no change in the four stock prices within the time interval Δt is positive.

The parameters $b_i, d_i, i = 1,2,3,4.,$ define the rate at which stocks experience individual gains or losses respectively.

The parameters $\alpha_{j,k}, \beta_{j,k}, \gamma_{j,k}, \eta_{j,k}, \delta_{j,k}, \xi_{j,k}, \varphi_{j,k,l}, \theta_{j,k,l}, \lambda_{j,k,l}, \psi_{j,k,l}, \pi_{j,k,l,m}$ for $j,k,l,m = 1,2$ defines the rate at which stocks experience simultaneous losses and/or gains respectively with each parameter depending on t.

For example, $b_i S_i \Delta t$ is the probability that stock i gains one unit in the time interval Δt.

The change involving S_1 and S_2 simultaneously is denoted by $\alpha_{j,k}$. $\beta_{j,k}$ denotes the simultaneous change involving S_1 and S_3. $\gamma_{j,k}$ denotes the simultaneous change involving S_1 and S_4. $\eta_{j,k}$ denotes the simultaneous change involving S_2 and S_3. $\delta_{j,k}$ denotes simultaneous change involving S_2 and S_4. $\xi_{j,k}$ denotes the simultaneous change involving S_3 and S_4. $\varphi_{j,k,l}$ denotes the simultaneous change involving S_1, S_2 and S_3. $\theta_{j,k,l}$ denotes the simultaneous change involving S_1, S_2 and S_4. $\lambda_{j,k,l}$ denotes the simultaneous change involving S_1, S_3 and S_4. $\psi_{j,k,l}$ denotes the simultaneous change involving S_2, S_3 and S_4. While $\pi_{j,k,l,m}$ denotes the simultaneous change involving S_1, S_2, S_3 and S_4.. For example, α_{21} represents the change involving the two stocks S_1 and S_2 in which S_1 gains while S_2 loses. β_{12} represents the change involving the two stocks S_1 and S_3 in which S_1 loses while S_3 gains. η_{22} denotes the change involving the two stocks S_2 and S_3 simultaneously in which both stocks gain. θ_{221} denotes the change involving the three stocks S_1, S_2 and S_4 simultaneously in which stocks S_1 and S_2 gain while S_4 loses. Finally, π_{1221} denotes the simultaneous change involving all four stocks in which stocks S_1 and S_4 experience loss while stocks S_2 and S_3 experience gain. In all the cases, the subscript 1 or 2 represent loss of one unit or gain of one unit respectively. It should be noted that $\sum_{i=1}^{81} p_i = 1$.

Using the above representations for p_i and ΔS_i the expectation vector is as follows:

$$E(\Delta S) = \sum_{i=1}^{81} p_i \Delta S_i = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \quad (3.1)$$

$$\begin{aligned}
f_1 &= (d_1+b_1)S_1 + (\alpha_{11}+\alpha_{12}+\alpha_{21}+\alpha_{22})S_1S_2 + (\beta_{11}+\beta_{12}+\beta_{21}+\beta_{22})S_1S_3 \\
&+ (\gamma_{11}+\gamma_{12}+\gamma_{21}+\gamma_{22})S_1S_4 + \begin{pmatrix}\varphi_{111}+\varphi_{112}+\varphi_{121}+\varphi_{122} \\ +\varphi_{211}+\varphi_{212}+\varphi_{221}+\varphi_{222}\end{pmatrix}S_1S_2S_3 \\
&+ \begin{pmatrix}\theta_{111}+\theta_{112}+\theta_{121}+\theta_{122} \\ +\theta_{211}+\theta_{212}+\theta_{221}+\theta_{222}\end{pmatrix}S_1S_2S_4 + \begin{pmatrix}\lambda_{111}+\lambda_{112}+\lambda_{121}+\lambda_{122} \\ +\lambda_{211}+\lambda_{212}+\lambda_{221}+\lambda_{222}\end{pmatrix}S_1S_3S_4 \\
&+ \begin{pmatrix}\pi_{1111}+\pi_{1112}+\pi_{1121}+\pi_{1122}+\pi_{1211}+\pi_{1212}+\pi_{1221}+\pi_{1222} \\ +\pi_{2111}+\pi_{2112}+\pi_{2121}+\pi_{2122}+\pi_{2211}+\pi_{2212}+\pi_{2221}+\pi_{2222}\end{pmatrix}S_1S_2S_3S_4
\end{aligned} \tag{3.2}$$

f_1 represents the totality of the likelihood of change involving stock S_1.

$$\begin{aligned}
f_2 &= (d_2+b_2)S_2 + (\alpha_{11}+\alpha_{12}+\alpha_{21}+\alpha_{22})S_1S_2 + (\eta_{11}+\eta_{12}+\eta_{21}+\eta_{22})S_2S_3 \\
&+ (\delta_{11}+\delta_{12}+\delta_{21}+\delta_{22})S_2S_4 + \begin{pmatrix}\varphi_{111}+\varphi_{112}+\varphi_{121}+\varphi_{122} \\ +\varphi_{211}+\varphi_{212}+\varphi_{221}+\varphi_{222}\end{pmatrix}S_1S_2S_3 \\
&+ \begin{pmatrix}\theta_{111}+\theta_{112}+\theta_{121}+\theta_{122} \\ +\theta_{211}+\theta_{212}+\theta_{221}+\theta_{222}\end{pmatrix}S_1S_2S_4 + \begin{pmatrix}\psi_{111}+\psi_{112}+\psi_{121}+\psi_{122} \\ +\psi_{211}+\psi_{212}+\psi_{221}+\psi_{222}\end{pmatrix}S_2S_3S_4 \\
&+ \begin{pmatrix}\pi_{1111}+\pi_{1112}+\pi_{1121}+\pi_{1122}+\pi_{1211}+\pi_{1212}+\pi_{1221}+\pi_{1222} \\ +\pi_{2111}+\pi_{2112}+\pi_{2121}+\pi_{2122}+\pi_{2211}+\pi_{2212}+\pi_{2221}+\pi_{2222}\end{pmatrix}S_1S_2S_3S_4
\end{aligned} \tag{3.3}$$

f_2 represents the totality of the likelihood of change involving stock S_2.

$$\begin{aligned}
f_3 &= (d_3+b_3)S_3 + (\beta_{11}+\beta_{12}+\beta_{21}+\beta_{22})S_1S_3 + (\eta_{11}+\eta_{12}+\eta_{21}+\eta_{22})S_2S_3 \\
&+ (\xi_{11}+\xi_{12}+\xi_{21}+\xi_{22})S_3S_4 + \begin{pmatrix}\varphi_{111}+\varphi_{112}+\varphi_{121}+\varphi_{122} \\ +\varphi_{211}+\varphi_{212}+\varphi_{221}+\varphi_{222}\end{pmatrix}S_1S_2S_3 \\
&+ \begin{pmatrix}\lambda_{111}+\lambda_{112}+\lambda_{121}+\lambda_{122} \\ +\lambda_{211}+\lambda_{212}+\lambda_{221}+\lambda_{222}\end{pmatrix}S_1S_3S_4 + \begin{pmatrix}\psi_{111}+\psi_{112}+\psi_{121}+\psi_{122} \\ +\psi_{211}+\psi_{212}+\psi_{221}+\psi_{222}\end{pmatrix}S_2S_3S_4 \\
&+ \begin{pmatrix}\pi_{1111}+\pi_{1112}+\pi_{1121}+\pi_{1122}+\pi_{1211}+\pi_{1212}+\pi_{1221}+\pi_{1222} \\ +\pi_{2111}+\pi_{2112}+\pi_{2121}+\pi_{2122}+\pi_{2211}+\pi_{2212}+\pi_{2221}+\pi_{2222}\end{pmatrix}S_1S_2S_3S_4
\end{aligned} \tag{3.4}$$

f_3 represents the totality of the likelihood of change involving stock S_3.

$$\begin{aligned}
f_4 &= (d_4+b_4)S_4 + (\gamma_{11}+\gamma_{12}+\gamma_{21}+\gamma_{22})S_1S_4 + (\delta_{11}+\delta_{12}+\delta_{21}+\delta_{22})S_2S_4 \\
&+ (\xi_{11}+\xi_{12}+\xi_{21}+\xi_{22})S_3S_4 + \begin{pmatrix}\theta_{111}+\theta_{112}+\theta_{121}+\theta_{122} \\ +\theta_{211}+\theta_{212}+\theta_{221}+\theta_{222}\end{pmatrix}S_1S_2S_4 \\
&+ \begin{pmatrix}\lambda_{111}+\lambda_{112}+\lambda_{121}+\lambda_{122} \\ +\lambda_{211}+\lambda_{212}+\lambda_{221}+\lambda_{222}\end{pmatrix}S_1S_3S_4 + \begin{pmatrix}\psi_{111}+\psi_{112}+\psi_{121}+\psi_{122} \\ +\psi_{211}+\psi_{212}+\psi_{221}+\psi_{222}\end{pmatrix}S_2S_3S_4 \\
&+ \begin{pmatrix}\pi_{1111}+\pi_{1112}+\pi_{1121}+\pi_{1122}+\pi_{1211}+\pi_{1212}+\pi_{1221}+\pi_{1222} \\ +\pi_{2111}+\pi_{2112}+\pi_{2121}+\pi_{2122}+\pi_{2211}+\pi_{2212}+\pi_{2221}+\pi_{2222}\end{pmatrix}S_1S_2S_3S_4
\end{aligned} \tag{3.5}$$

f_4 represents the totality of the likelihood of change involving stock S_4

Furthermore, from (3.1) and neglecting terms of order $(\Delta S)^2$, we have

$$E\left(\Delta S\left(\Delta S\right)^T\right)=\begin{bmatrix} f_1 f_1 & f_1 f_2 & f_1 f_3 & f_1 f_4 \\ f_2 f_1 & f_2 f_2 & f_2 f_3 & f_2 f_4 \\ f_3 f_1 & f_3 f_2 & f_3 f_3 & f_3 f_4 \\ f_4 f_1 & f_4 f_2 & f_4 f_3 & f_4 f_4 \end{bmatrix} \tag{3.6}$$

where

$f_1 f_1 \Rightarrow$ likelihood of change in price occurring in S_1 only = dS_1
$f_2 f_2 \Rightarrow$ likelihood of change in price occurring in S_2 only = dS_2
$f_3 f_3 \Rightarrow$ likelihood of change in price occurring in S_3 only = dS_3
$f_4 f_4 \Rightarrow$ likelihood of change in price occurring in S_4 only = dS_4
$f_1 f_2 \Rightarrow$ likelihood of change in price occurring in S_1 and S_2 only = dS_1S_2
$f_1 f_3 \Rightarrow$ likelihood of change in price occurring in S_1 and S_3 only = dS_1S_3
$f_1 f_4 \Rightarrow$ likelihood of change in price occurring in S_1 and S_4 only = dS_1S_4
$f_2 f_3 \Rightarrow$ likelihood of change in price occurring in S_2 and S_3 only = dS_2S_3
$f_2 f_4 \Rightarrow$ likelihood of change in price occurring in S_2 and S_4 only = dS_2S_4
$f_3 f_4 \Rightarrow$ likelihood of change in price occurring in S_3 and S_4 only = dS_3S_4

Inserting the following in (3.6),

$$dS_1 = (d_1 + b_1)S_1 \tag{3.7}$$

$$dS_2 = (d_2 + b_2)S_2, \tag{3.8}$$

$$dS_3 = (d_3 + b_3)S_3, \tag{3.9}$$

$$dS_4 = \left(d_4 + b_4\right)S_4 \tag{3.10}$$

$$dS_1S_2 = \left(\alpha_{11} + \alpha_{12} + \alpha_{21} + \alpha_{22}\right)S_1S_2, \tag{3.11}$$

$$dS_1S_3 = \left(\beta_{11} + \beta_{12} + \beta_{21} + \beta_{22}\right)S_1S_3, \tag{3.12}$$

$$dS_1S_4 = \left(\gamma_{11} + \gamma_{12} + \gamma_{21} + \gamma_{22}\right)S_1S_4. \tag{3.13}$$

$$dS_2S_3 = \left(\eta_{11} + \eta_{12} + \eta_{21} + \eta_{22}\right)S_2S_3 \tag{3.14}$$

$$dS_2S_4 = \left(\delta_{11} + \delta_{12} + \delta_{21} + \delta_{22}\right) S_2S_4 \tag{3.15}$$

$$dS_3S_4 = \left(\xi_{11} + \xi_{12} + \xi_{21} + \xi_{22}\right) S_3S_4 \tag{3.16}$$

$$dS_1S_2S_3 = \left(\varphi_{111} + \varphi_{112} + \varphi_{121} + \varphi_{122} + \varphi_{211} + \varphi_{212} + \varphi_{221} + \varphi_{222}\right) S_1S_2S_3 \tag{3.17}$$

$$dS_1S_2S_4 = \left(\theta_{111} + \theta_{112} + \theta_{121} + \theta_{122} + \theta_{211} + \theta_{212} + \theta_{221} + \theta_{222}\right) S_1S_2S_4 \tag{3.18}$$

$$dS_1S_3S_4 = \left(\lambda_{111} + \lambda_{112} + \lambda_{121} + \lambda_{122} + \lambda_{211} + \lambda_{212} + \lambda_{221} + \lambda_{222}\right) S_1S_3S_4 \tag{3.19}$$

$$dS_2S_3S_4 = \left(\psi_{111} + \psi_{112} + \psi_{121} + \psi_{122} + \psi_{211} + \psi_{212} + \psi_{221} + \psi_{222}\right) S_2S_3S_4 \tag{3.20}$$

$$dS_1S_2S_3S_4 = \begin{pmatrix} \pi_{1111} + \pi_{1112} + \pi_{1121} + \pi_{1122} + \pi_{1211} + \pi_{1212} + \pi_{1221} + \pi_{1222} \\ +\pi_{2111} + \pi_{2112} + \pi_{2121} + \pi_{2122} + \pi_{2211} + \pi_{2212} + \pi_{2221} + \pi_{2222} \end{pmatrix} S_1S_2S_3S_4 \tag{3.21}$$

Then the covariance matrix is as follows:

$$E(\Delta S(\Delta S)^T) = \begin{bmatrix} dS_1 & dS_1S_2 & dS_1S_3 & dS_1S_4 \\ dS_2S_1 & dS_2 & dS_2S_3 & dS_2S_4 \\ dS_3S_1 & dS_3S_2 & dS_3 & dS_3S_4 \\ dS_4S_1 & dS_4S_2 & dS_4S_3 & dS_4 \end{bmatrix} \tag{3.22}$$

The covariance matrix represents the volatility coefficient of the SDE. Clearly, the covariance matrix is a positive definite symmetric matrix and hence has a positive definite square root. Therefore adopting the first modeling procedure as in Allen (2003), we then have the SDE.

$$dS(t) = \mu(t, S_1, S_2, S_3, S_4)dt + B(t, S_1, S_2, S_3, S_4)dW(t) \tag{3.23}$$

which is an initial value problem and where

$$\mu(t, S_1, S_2, S_3, S_4) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}, \tag{3.24}$$

dW(t) is a four-dimensional matrix and

$$B(t, S_1, S_2, S_3, S_4) = \begin{bmatrix} dS_1 & dS_1S_2 & dS_1S_3 & dS_1S_4 \\ dS_2S_1 & dS_2 & dS_2S_3 & dS_2S_4 \\ dS_3S_1 & dS_3S_2 & dS_3 & dS_3S_4 \\ dS_4S_1 & dS_4S_2 & dS_4S_3 & dS_4 \end{bmatrix}^{1/2} \tag{3.25}$$

From Table 2.1 above, a checklist of instances of loss, gain or unchanged or stable were noted and marked out as shown below in Table 3.2: A stock price could lose (L), gain (G) or remain unchanged or stable (N) in the day considered. This was cross checked for each stock and recorded, with a portion shown in Table 3.2 below.

Table 3.2. A checklist of occurrences of gain, stable or loss in the prices of the 60 days

Day	S1			S2			S3			S4			Summary
	L	N	G	L	N	G	L	N	G	L	N	G	
1													
2			√		√		√					√	[1,0,-1,1]
3	√				√		√				√		[-1,0,-1,0]
4	√				√		√				√		[-1,0,-1,0]
5			√		√			√		√			[1,0,0,-1]
6			√		√				√		√		[1,0,1,0]
7			√		√			√			√		[1,0,0,0]
8	√				√			√		√			[-1,0,0,-1]
9	√				√			√		√			[-1,0,0,-1]
10	√				√		√			√			[-1,0,-1,-1]
11			√	√					√			√	[1,-1,1,1]
12		√				√	√				√		[0,1,-1,0]
13		√			√				√		√		[0,0,1,0]
14		√			√			√			√		[0,0,0,0]
15		√			√			√				√	[0,0,0,1]

The summary column captures how the prices of each stock vary in relation to the previous day. For example, (0,0,0,0) in the summary column indicates that the price of stock 1, stock 2, stock 3 and stock 4 did not change; (-1,1,0,0) shows that stock1 lost, stock 2 gained while stocks 3 and 4 remained stable and so on.

4 Results and Discussion

In this Section, the drift and the volatility coefficients of the stochastic differential equations are determined. Due the difficulty in solving stochastic differential equations, the multi-dimensional Euler-Maruyama method is employed to solve and simulate stock prices. The results are also discussed.

4.1 The drift and volatility coefficients

From Table 3.2 above the probability of each occurrence was determined by

$$p_i = \frac{number\ of\ occurence}{total\ number\ of\ times\ the\ events\ could\ occur\ (81)}$$

For example, the event that the change in the stock prices is [0, 1,-1, 0] which occurred 2 times is $\frac{2}{81} = 0.0247$. The probabilities for the other events were determined and tabulated as shown in Table 4.1.

Table 4.1. Pattern of change in price and their probabilities

S/N	Change in Stock Price $\Delta S = [\Delta S_1, \Delta S_2, \Delta S_3, \Delta S_4]^T$	No. of occurrence	Probabilities
1	[-1,-1,-1,-1]	0	0.0000
2	[-1,-1,-1,0]	0	0.0000
3	[-1,-1,-1,1]	0	0.0000
4	[-1,-1,0,-1]	0	0.0000
5	[-1,-1,0,0]	0	0.0000
6	[-1,-1,0, 1]	0	0.0000
7	[-1,-1,1,-1]	1	0.0123
8	[-1,-1,1, 0]	0	0.0000
9	[-1,-1,1, 1]	0	0.0000
10	[-1,0,-1,-1]	1	0.0123
11	[-1,0,-1, 0]	3	0.0370
12	[-1,0,-1,1]	0	0.0000
13	[-1,0,0,-1]	2	0.0247
14	[-1,0,0,0]	2	0.0247
15	[-1,0,0,1]	3	0.0370
16	[-1,0,1,-1]	0	0.0000
17	[-1,0,1,0]	0	0.0000
18	[-1,0,1,1]	0	0.0000
19	[-1,1,-1,-1]	1	0.0123
20	[-1,1,-1,0]	0	0.0000
21	[-1,1,-1,1]	0	0.0000
22	[-1,1,0,-1]	0	0.0000
23	[-1,1,0,0]	0	0.0000
24	[-1,1,0,1]	0	0.0000
25	[-1,1,1,-1]	1	0.0123
26	[-1,1,1,0]	1	0.0123
27	[-1,1,1,1]	0	0.0000
28	[0,-1,-1,-1]	0	0.0000
29	[0,-1,-1,0]	0	0.0000
30	[0,-1,-1,1]	0	0.0000
31	[0,-1,0,-1]	0	0.0000
32	[0,-1,0,0]	1	0.0123
33	[0,-1,0,1]	0	0.0000
34	[0,-1,1,-1]	0	0.0000
35	[0,-1,1,0]	1	0.0123
36	[0,-1,1,1]	0	0.0000
37	[0,0,-1,-1]	0	0.0000
38	[0,0,-1,0]	2	0.0247
39	[0,0,-1,1]	0	0.0000
40	[0,0,0,-1]	1	0.0123
41	[0,0,0,0]	14	0.1728
42	[0,0,0,1]	4	0.0494
43	[0,0,1,-1]	1	0.0123
44	[0,0,1,0]	4	0.0494
45	[0,0,1,1]	0	0.0000
46	[0,1,-1,-1]	0	0.0000
47	[0,1,-1,0]	2	0.0247
48	[0,1,-1,1]	0	0.0000
49	[0,1,0,-1]	0	0.0000
50	[0,1,0,0]	1	0.0123

S/N	**Change in Stock Price** $\Delta S = [\Delta S_1, \Delta S_2, \Delta S_3, \Delta S_4]^T$	**No. of occurrence**	**Probabilities**
51	[0,1,0,1]	0	0.0000
52	[0,1,1,-1]	0	0.0000
53	[0,1,1,0]	0	0.0000
54	[0,1,1,1]	0	0.0000
55	[1,-1,-1,-1]	0	0.0000
56	[1,-1,-1,0]	1	0.0123
57	[1,-1,-1,1]	0	0.0000
58	[1,-1,0,-1]	0	0.0000
59	[1,-1,0,0]	1	0.0123
60	[1,-1,0,1]	0	0.0000
61	[1,-1,1,-1]	0	0.0000
62	[1,-1,1,0]	0	0.0000
63	[1,-1,1,1]	1	0.0123
64	[1,0,-1,-1]	0	0.0000
65	[1,0,-1,0]	1	0.0123
66	[1,0,-1,1]	1	0.0123
67	[1,0,0,-1]	2	0.0247
68	[1,0,0,0]	3	0.0370
69	[1,0,0,1]	1	0.0123
70	[1,0,1,-1]	0	0.0123
71	[1,0,1,0]	2	0.0247
72	[1,0,1,1]	0	0.0000
73	[1,1,-1,-1]	0	0.0000
74	[1,1,-1,0]	0	0.0000
75	[1,1,-1,1]	0	0.0000
76	[1,1,0,-1]	0	0.0000
77	[1,1,0,0]	0	0.0000
78	[1,1,0,1]	0	0.0000
79	[1,1,1,-1]	0	0.0000
80	[1,1,1,0]	0	0.0000
81	[1,1,1,1]	0	0.0000

From Table 4.1 above relevant variables were computed as follows:

$$\begin{aligned} f_1 &= (d_1 + b_1)S_1 + (\alpha_{11} + \alpha_{12} + \alpha_{21} + \alpha_{22})S_1S_2 + (\beta_{11} + \beta_{12} + \beta_{21} + \beta_{22})S_1S_3 \\ &+ (\gamma_{11} + \gamma_{12} + \gamma_{21} + \gamma_{22})S_1S_4 + \begin{pmatrix} \varphi_{111} + \varphi_{112} + \varphi_{121} + \varphi_{122} \\ +\varphi_{211} + \varphi_{212} + \varphi_{221} + \varphi_{222} \end{pmatrix} S_1S_2S_3 \\ &+ \begin{pmatrix} \theta_{111} + \theta_{112} + \theta_{121} + \theta_{122} \\ +\theta_{211} + \theta_{212} + \theta_{221} + \theta_{222} \end{pmatrix} S_1S_2S_4 + \begin{pmatrix} \lambda_{111} + \lambda_{112} + \lambda_{121} + \lambda_{122} \\ +\lambda_{211} + \lambda_{212} + \lambda_{221} + \lambda_{222} \end{pmatrix} S_1S_3S_4 \\ &+ \begin{pmatrix} \pi_{1111} + \pi_{1112} + \pi_{1121} + \pi_{1122} + \pi_{1211} + \pi_{1212} + \pi_{1221} + \pi_{1222} \\ +\pi_{2111} + \pi_{2112} + \pi_{2121} + \pi_{2122} + \pi_{2211} + \pi_{2212} + \pi_{2221} + \pi_{2222} \end{pmatrix} S_1S_2S_3S_4 \end{aligned}$$

$$\begin{aligned} f_1 &= 0.0247 + 0.0370 + 0.0123 + 0.0370 + 0.0123 + 0.0247 + 0.0247 + 0.0370 + 0.0247 + 0.0123 + 0.0123 + 0.0123 \\ &+ 0.0123 + 0.0123 + 0.0123 + 0.0123 + 0.0123 + 0.0123 = 0.03451 \end{aligned}$$

$$
\begin{aligned}
f_2 &= (d_2+b_2)S_2 + (\alpha_{11}+\alpha_{12}+\alpha_{21}+\alpha_{22})S_1S_2 + (\eta_{11}+\eta_{12}+\eta_{21}+\eta_{22})S_2S_3 \\
&+ (\delta_{11}+\delta_{12}+\delta_{21}+\delta_{22})S_2S_4 + \begin{pmatrix} \varphi_{111}+\varphi_{112}+\varphi_{121}+\varphi_{122} \\ +\varphi_{211}+\varphi_{212}+\varphi_{221}+\varphi_{222} \end{pmatrix} S_1S_2S_3 \\
&+ \begin{pmatrix} \theta_{111}+\theta_{112}+\theta_{121}+\theta_{122} \\ +\theta_{211}+\theta_{212}+\theta_{221}+\theta_{222} \end{pmatrix} S_1S_2S_4 + \begin{pmatrix} \psi_{111}+\psi_{112}+\psi_{121}+\psi_{122} \\ +\psi_{211}+\psi_{212}+\psi_{221}+\psi_{222} \end{pmatrix} S_2S_3S_4 \\
&+ \begin{pmatrix} \pi_{1111}+\pi_{1112}+\pi_{1121}+\pi_{1122}+\pi_{1211}+\pi_{1212}+\pi_{1221}+\pi_{1222} \\ +\pi_{2111}+\pi_{2112}+\pi_{2121}+\pi_{2122}+\pi_{2211}+\pi_{2212}+\pi_{2221}+\pi_{2222} \end{pmatrix} S_1S_2S_3S_4
\end{aligned}
$$

$$f_2 = 0.0123+0.0123+0.0123+0.0123+0.0247+0.0123+0.0123+0.0123+0.0123+0.0123+0.0123 = 0.1477$$

$$
\begin{aligned}
f_3 &= (d_3+b_3)S_3 + (\beta_{11}+\beta_{12}+\beta_{21}+\beta_{22})S_1S_3 + (\eta_{11}+\eta_{12}+\eta_{21}+\eta_{22})S_2S_3 \\
&+ (\xi_{11}+\xi_{12}+\xi_{21}+\xi_{22})S_3S_4 + \begin{pmatrix} \varphi_{111}+\varphi_{112}+\varphi_{121}+\varphi_{122} \\ +\varphi_{211}+\varphi_{212}+\varphi_{221}+\varphi_{222} \end{pmatrix} S_1S_2S_3 \\
&+ \begin{pmatrix} \lambda_{111}+\lambda_{112}+\lambda_{121}+\lambda_{122} \\ +\lambda_{211}+\lambda_{212}+\lambda_{221}+\lambda_{222} \end{pmatrix} S_1S_3S_4 + \begin{pmatrix} \psi_{111}+\psi_{112}+\psi_{121}+\psi_{122} \\ +\psi_{211}+\psi_{212}+\psi_{221}+\psi_{222} \end{pmatrix} S_2S_3S_4 \\
&+ \begin{pmatrix} \pi_{1111}+\pi_{1112}+\pi_{1121}+\pi_{1122}+\pi_{1211}+\pi_{1212}+\pi_{1221}+\pi_{1222} \\ +\pi_{2111}+\pi_{2112}+\pi_{2121}+\pi_{2122}+\pi_{2211}+\pi_{2212}+\pi_{2221}+\pi_{2222} \end{pmatrix} S_1S_2S_3S_4
\end{aligned}
$$

$$
\begin{aligned}
f_3 &= 0.0247+0.0494+0.0370+0.0123+0.0247+0.0123+0.0247+0.0123+0.0123+0.0123+0.0123+0.0123+0.0123 \\
&\quad 0.0123+0.0123+0.0123 = 0.2958
\end{aligned}
$$

$$
\begin{aligned}
f_4 &= (d_4+b_4)S_4 + (\gamma_{11}+\gamma_{12}+\gamma_{21}+\gamma_{22})S_1S_4 + (\delta_{11}+\delta_{12}+\delta_{21}+\delta_{22})S_2S_4 \\
&+ (\xi_{11}+\xi_{12}+\xi_{21}+\xi_{22})S_3S_4 + \begin{pmatrix} \theta_{111}+\theta_{112}+\theta_{121}+\theta_{122} \\ +\theta_{211}+\theta_{212}+\theta_{221}+\theta_{222} \end{pmatrix} S_1S_2S_4 \\
&+ \begin{pmatrix} \lambda_{111}+\lambda_{112}+\lambda_{121}+\lambda_{122} \\ +\lambda_{211}+\lambda_{212}+\lambda_{221}+\lambda_{222} \end{pmatrix} S_1S_3S_4 + \begin{pmatrix} \psi_{111}+\psi_{112}+\psi_{121}+\psi_{122} \\ +\psi_{211}+\psi_{212}+\psi_{221}+\psi_{222} \end{pmatrix} S_2S_3S_4 \\
&+ \begin{pmatrix} \pi_{1111}+\pi_{1112}+\pi_{1121}+\pi_{1122}+\pi_{1211}+\pi_{1212}+\pi_{1221}+\pi_{1222} \\ +\pi_{2111}+\pi_{2112}+\pi_{2121}+\pi_{2122}+\pi_{2211}+\pi_{2212}+\pi_{2221}+\pi_{2222} \end{pmatrix} S_1S_2S_3S_4
\end{aligned}
$$

$$f_4 = 0.0123+0.0494+0.0247+0.0370+0.0247+0.0123+0.0123+0.0123+0.0123+0.0123+0.0123+0.0123+0.0123 = 0.2465$$

Hence, the expectation vector

$$E(\Delta S) = \sum_{i=1}^{81} p_i \Delta S_i = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} 0.3451 \\ 0.1477 \\ 0.2958 \\ 0.2465 \end{bmatrix}$$

The likelihood of the change in the price occurring with stockS_1only is

$$dS_1 = (d_1 + b_1) S_1 = 0.0247 + 0.0370 = 0.0617$$

The likelihood of the change occurring in the price of stock S_2 only is

$$dS_2 = (d_2 + b_2) S_2 = 0.0123 + 0.0123 = 0.0246$$

The likelihood of the change occurring in the price of stock S_3 only is

$$dS_3 = (d_3 + b_3) S_3 = 0.0247 + 0.0494 = 0.0741$$

The likelihood of the change occurring in the price of stock S_4 only is

$$dS_4 = (d_4 + b_4) S_4 = 0.0123 + 0.0494 = 0.0617$$

The likelihood of change in the prices of stocks$S_1$and S_2 only is

$$dS_1S_2 = (\alpha_{11} + \alpha_{12} + \alpha_{21} + \alpha_{22}) S_1S_2 = 0.0123$$

The likelihood of change in the prices of stocks $S_1$and S_3only is

$$dS_1S_3 = (\beta_{11} + \beta_{12} + \beta_{21} + \beta_{22}) S_1S_3$$
$$= 0.0370 + 0.0123 + 0.0247 = 0.0740$$

The likelihood of change in the prices of stocks $S_1$and S_4only is

$$dS_1S_4 = (\gamma_{11} + \gamma_{12} + \gamma_{21} + \gamma_{22}) S_1S_4$$
$$= 0.0247 + 0.0370 + 0.0247 + 0.0123 = 0.0987$$

The likelihood of change in the prices of stocks S_2 and S_3 only is

$$dS_2S_3 = (\eta_{11} + \eta_{12} + \eta_{21} + \eta_{22}) S_2S_3$$
$$= 0.0123 + 0.0247 = 0.0370$$

The likelihood of change in the prices of stocks S_2 and S_4 only is

$$dS_2S_4 = (\delta_{11} + \delta_{12} + \delta_{21} + \delta_{22}) S_2S_4 = 0.0000$$

The likelihood of change in the prices of stocks S_3 and S_4 only is

$$dS_3S_4 = (\xi_{11} + \xi_{12} + \xi_{21} + \xi_{22}) S_3S_4 = 0.0123$$

The likelihood of change in the prices of stocks S_1, S_2 and $S_3$is

$$dS_1S_2S_3 = (\varphi_{111} + \varphi_{112} + \varphi_{121} + \varphi_{122} + \varphi_{211} + \varphi_{212} + \varphi_{221} + \varphi_{222}) S_1S_2S_3$$
$$= 0.0123 + 0.0123 = 0.0246$$

The likelihood of change in the prices of stocks S_1, S_2 and S_4 is

$$dS_1S_2S_4 = \left(\theta_{111} + \theta_{112} + \theta_{121} + \theta_{122} + \theta_{211} + \theta_{212} + \theta_{221} + \theta_{222}\right) S_1S_2S_4 = 0.0000$$

The likelihood of change in the prices of stocks S_1, S_3 and S_4 is

$$dS_1S_3S_4 = \left(\lambda_{111} + \lambda_{112} + \lambda_{121} + \lambda_{122} + \lambda_{211} + \lambda_{212} + \lambda_{221} + \lambda_{222}\right) S_1S_3S_4$$
$$= 0.0123 + 0.0123 = 0.0246$$

The likelihood of change in the prices of stocks S_2, S_3 and S_4 is

$$dS_2S_3S_4 = \left(\psi_{111} + \psi_{112} + \psi_{121} + \psi_{122} + \psi_{211} + \psi_{212} + \psi_{221} + \psi_{222}\right) S_2S_3S_4 = 0.0000$$

The likelihood of change in the prices of the four stocks S_1, S_2, S_3 and S_4 concurrently is

$$dS_1S_2S_3S_4 = \begin{pmatrix} \pi_{1111} + \pi_{1112} + \pi_{1121} + \pi_{1122} + \pi_{1211} + \pi_{1212} + \pi_{1221} + \pi_{1222} \\ +\pi_{2111} + \pi_{2112} + \pi_{2121} + \pi_{2122} + \pi_{2211} + \pi_{2212} + \pi_{2221} + \pi_{2222} \end{pmatrix} S_1S_2S_3S_4$$
$$= 0.0123 + 0.0123 + 0.0123 + 0.0123 = 0.0492$$

Consequently, the covariance matrix is

$$E(\Delta S(\Delta S)^T) = \begin{bmatrix} dS_1 & dS_1S_2 & dS_1S_3 & dS_1S_4 \\ dS_2S_1 & dS_2 & dS_2S_3 & dS_2S_4 \\ dS_3S_1 & dS_3S_2 & dS_3 & dS_3S_4 \\ dS_4S_1 & dS_4S_2 & dS_4S_3 & dS_4 \end{bmatrix} = \begin{bmatrix} 0.0617 & 0.0123 & 0.0740 & 0.0987 \\ 0.0123 & 0.0246 & 0.0370 & 0.0000 \\ 0.0740 & 0.0370 & 0.0741 & 0.0123 \\ 0.0987 & 0.0000 & 0.0123 & 0.0617 \end{bmatrix}$$

The resulting stochastic differential equation is therefore:

$$dS(t) = \mu(t, S_1, S_2, S_3, S_4)dt + B(t, S_1, S_2, S_3, S_4)dW(t)$$

$$dS(t) = \begin{bmatrix} 0.3451 \\ 0.1477 \\ 0.2958 \\ 0.2465 \end{bmatrix} dt + \begin{bmatrix} 0.0617 & 0.0123 & 0.0740 & 0.0987 \\ 0.0123 & 0.0246 & 0.0370 & 0.0000 \\ 0.0740 & 0.0370 & 0.0741 & 0.0123 \\ 0.0987 & 0.0000 & 0.0123 & 0.0617 \end{bmatrix}^{1/2} dW(t) \tag{4.1}$$

$$dS(t) = \begin{bmatrix} 0.3451 \\ 0.1477 \\ 0.2958 \\ 0.2465 \end{bmatrix} dt + \begin{bmatrix} 0.2484 & 0.1109 & 0.2720 & 0.3142 \\ 0.1109 & 0.1568 & 0.1924 & 0.0000 \\ 0.2720 & 0.1924 & 0.2722 & 0.1109 \\ 0.3142 & 0.0000 & 0.1109 & 0.2484 \end{bmatrix} dW(t) \tag{4.2}$$

Breaking down the matrices, we have the following system of equations

$$
\begin{aligned}
dS_1 &= 0.3451dt + 0.2484dW_1 + 0.1109dW_2 + 0.2720dW_3 + 0.3142dW_4 \\
dS_2 &= 0.1477dt + 0.1109dW_1 + 0.1568dW_2 + 0.1924dW_3 + 0.0000dW_4 \\
dS_3 &= 0.2958dt + 0.2720dW_1 + 0.1924dW_2 + 0.2722dW_3 + 0.1109dW_4 \\
dS_4 &= 0.2465dt + 0.3142dW_1 + 0.0000dW_2 + 0.1109dW_3 + 0.2484dW_4
\end{aligned} \tag{4.3}
$$

This is an initial value problem and the initial values for solving (4.3) are as follows:

$$
\begin{aligned}
y_{1_0} &= 20.71 \\
y_{2_0} &= 22.52 \\
y_{3_0} &= 21.02 \\
y_{4_0} &= 21.01
\end{aligned}
$$

4.2 Results

The SDE obtained was solved using the multi-dimensional Euler-Maruyama scheme for SDEs and was achieved through a MATLAB script file.

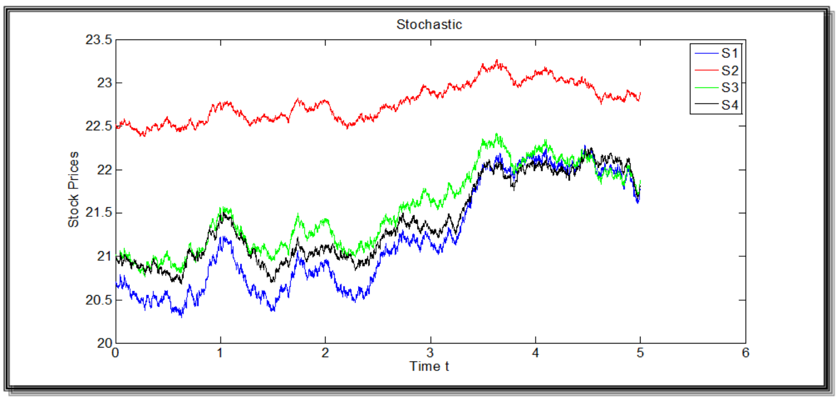

Fig. 4.1. Stock prices (in Naira) over a 5-month period

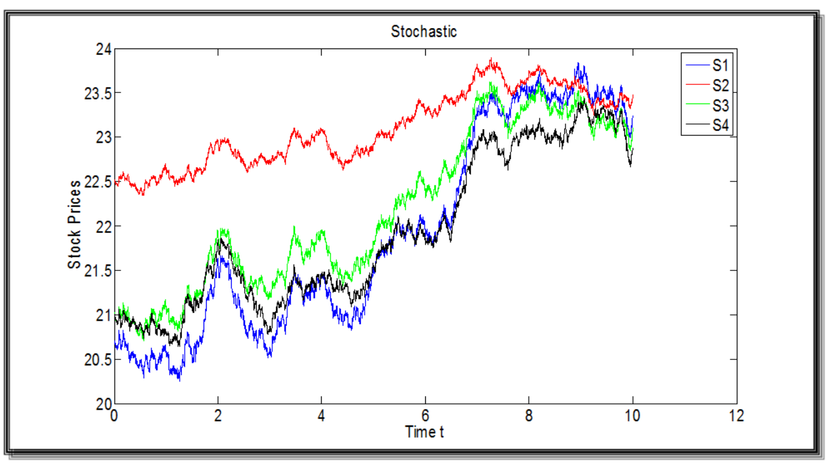

Fig. 4.2. Stock prices (in Naira) over a 10-month period

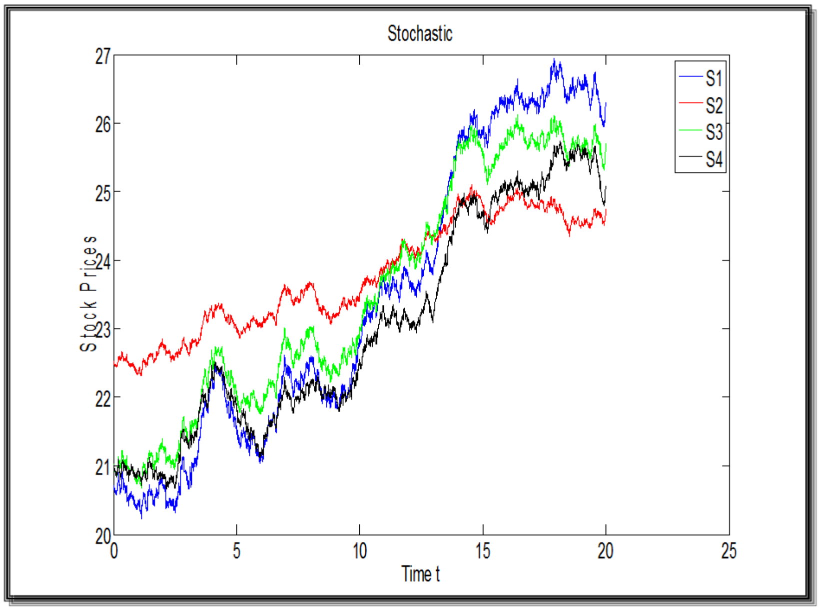

Fig. 4.3. Stock prices (in Naira) over a period of 20 months

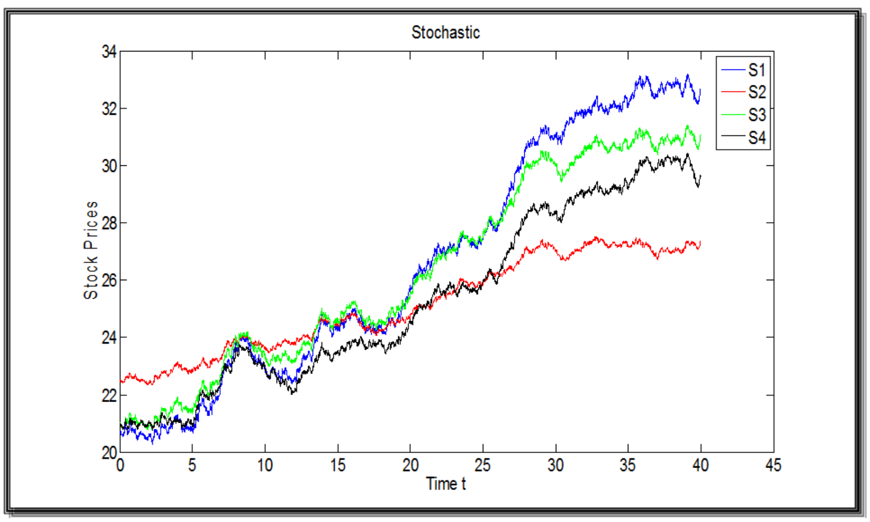

Fig. 4.4. Stock prices (in Naira) over a period of 40 months

5 Discussion of Results

Fig. 4.1 shows the different stock prices over a period of five months. The stock price of S_2 shows an insignificant increase over this time interval. However, the stock prices of S_1, S_3 and S_4 fluctuate significantly with a sharp increase after three months. These three stocks will give a high return on investment within this period.

Fig. 4.2 shows the different stock prices over a period of ten months. The four stock prices rise and fall with significant increase after six months and are all within the same price range towards the tenth month.

However, over a period of twenty months, as shown in Fig. 4.3, the stock S_1 is seen to give the highest return on investment. Likewise, stocks S_3 and S_4 experience drastic increase in price while S_2 remains somewhat stable.

Fig. 4.4 shows the different stock prices over a period of forty months. Still, as in Fig. 4.3, stock S_1 seems to give the best return on investment compared with stocks S_2, S_3 and S_4. The investor after observing the trend over longer period can invest in the stock that will yield the best returns. Our analysis enables us to compare as many stocks as possible in order to advise the investor on where best to make investment.

6 Summary, Conclusion and Recommendations

6.1 Summary

In this work, a stochastic model is developed for the dynamics of change in prices of selected stocks. Considering four stocks subjected to random influence by the market forces, we assume that in a small interval of time Δt, a stock price may change by losing one unit (-1), remaining stable (0) or gaining one unit (+1). Checking the probabilities of the gains and losses, the expectation vector and the covariance matrix are derived, resulting in a Stochastic Differential Equation. Due to the difficulty in solving most nonlinear Stochastic Differential Equations (SDEs) analytically, the Euler Maruyama Method for SDEs is used to solve and analyze the model with the aid of MATLAB software. The model is used to simulate prices over different or varied time intervals and predictions are made as regards the best stock(s) to invest in over a given time interval. This analysis enables the comparism of as many stocks as possible in order to advise the investor on where best to make investment.

6.2 Conclusion

The stock price of S_2 shows an insignificant increase over this time interval. However, the stock prices of S_1, S_3 and S_4 fluctuate significantly with a sharp increase after three months. These three stocks will give a high return on investment within this period.

The study further shows the different stock prices over a period of ten months. The four stock prices rise and fall with significant increase after six months and are all within the same price range towards the tenth month. However, over a period of twenty months, as shown in Fig. 4.3, the stock S_1 is seen to give the highest return on investment. Likewise, stocks S_3 and S_4 experience drastic increase in price while S_2 remains somewhat stable.

As could be seen in Fig. 4.4, the different stock prices over a period of forty months (as in Fig. 4.3) stock S_1 seems to give the best return on investment compared with stocks S_2, S_3 and S_4. The investor after observing the trend over longer period can invest in the stock that will yield the best returns. Our analysis enables us to compare as many stocks as possible in order to advise the investor on where best to make investment.

6.3 Recommendations

Looking at the complexity of computing the drift and volatility coefficients for the stochastic differential equation for four stocks (that is, $n = 4$), consideration can be given to formulating a computerized method of computation; thereby making calculation for $n > 4$ stocks easier to handle.

Stockbrokers could use this model to help their clients monitor their investments and in so doing, help in decision making in order to maximize profit and minimize loss. Consideration can also be given to solving the stochastic model with any other method and comparing the results. Further research may include analytic solution of our stochastic model.

Competing Interests

Authors have declared that no competing interests exist.

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